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# GENERALIZED INITIAL CONDITIONS FOR THE STOCHASTIC MODEL FOR POLLUTION AND DISSOLVED OXYGEN IN STREAMS

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# GENERALIZED INITIAL CONDITIONS FOR THE STOCHASTIC MODEL FOR POLLUTION AND DISSOLVED OXYGEN IN STREAMS

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# INTRODUCTION

## The General Problem

Today there is a tremendous volume of waste material that is being deposited daily into the streams and rivers throughout the United States. The waste material is a by-product of an industrial and population expansion and is increasing in volume and complexity daily. These wastes cannot all be treated and transformed into inert, non-toxic compounds prior to being let into the streams. In the last decade the problem received considerable attention and state and federal water-pollution laws have been enacted. When a regulatory agency wishes to restrict the quality of organic waste discharged into a body of water, it will need some criteria for judging the pollutants introduced into the stream. A sanitary engineer within the regulatory agency has several general methods at his disposal,

The first general method as explained by Kothandaraman (4) is an effluent standard which either requires specific degrees of treatment, usually in terms of biochemical oxygen demand (BOD), suspended solids, and coliform removal or which requires a limit on the concentrations of the various components of the effluent being released into the stream. In any case the effluent approach deals with the pollution problem indirectly as opposed to dealing directly with the effect the pollution has upon the stream. This standard has another shortcoming since it requires the same level of removal of BOD and solids from all sources of pollution, regardless of the volume of waste.

The second general method of pollution control as explained by Kothandaraman (4) is a stream standard preferably used in conjunction with the effluent standard mentioned above. The stream standard alone specifies that the stream quality be maintained at a certain acceptable level at all times. The advantage of this standard is that it directly deals with the water pollution problem by ensuring that the stream quality does not drop below a certain standard level at any time, regardless of the effluent or conditions of the stream. The disadvantage of this standard is that the level is usually set so high that waste treatment costs are prohibitive. The reason for the high standard is to ensure that the variation in the stream quality at no time falls below a certain "undesirable" level. This undesirable level might be established as a level not particularly good for fish. The parameter usually used in this method as a measure of the pollution from biodegradable waste is the stream's mean dissolved oxygen (DO) level.

The third general method of pollution control as explained by Thayer and Krutchkoff (6) is a probabilistic stream standard whereby the polluter is allowed to deposit organic waste materials that have higher BOD and lower DO values a certain proportion of the time than is normally allowed. This standard like the latter and unlike the effluent standard can be more realistic in protecting fish and usually better matches the stream standards with the actual stream requirement. For example to require that DO concentration never be less than 4 ppm may be difficult to satisfy. However, to require that the concentration not be less than 2 ppm more than 5% of the time and not be less than 4 ppm more than 40% of the time may be easier to satisfy. It could happen that this standard be set higher than the usual and thus would necessitate a higher degree of treatment. However, the important point is that the treatment degree or quality standard can more accurately fit the needs of the stream.

## Stream Characteristics and Parameters

Streams have the capacity to assimilate organic matter in addition to diluting it. This assimilative capacity helps a municipal sewage plant cut waste treatment costs. A major factor that streams depend on to reduce the amount of pollution is dissolved oxygen.

If sufficient oxygen is available in a stream, then the complex organic wastes are reduced to simpler compounds by aerobic reactions which can be measured. The complex carbon molecules react with the available oxygen to release carbon dioxide. Also, the complex nitrogen molecules react with oxygen yielding nitrites. These nitrites are the end-products of the aerobic decomposition process and are nonoffensive and harmless to humans and fish. They do, however, feed plant life and can cause problems by stimulating algae growth. Those problems will not be considered here.

The parameter usually used to measury the aerobic reactions is the biochemical oxygen demand (BOD). This measurement is made in a laboratory test of a stream-water sample. The amount of oxygen consumed in 5 days by the aerobic bacteria in stabilizing the organic matter is denoted as the BOD. The temperature, amounts of sample, and other conditions are specified in the test. The BOD parameter is measured in units of parts per million (ppm) or milligrams per liter (mg/l), since both units of measure are equivalent.

Along with the BOD is the other critical parameter, dissolved oxygen (DO), which is also measured in ppm or mg/l. An adequate supply of DO in a

stream is necessary for fish and for decomposing organic materials. If sufficient dissolved oxygen is not available to the fish, they will eventually suffocate. If the organic waste material does not have sufficient oxygen, anaerobic reactions will predominate and result in undesirable and lethal end-products. These reactions result in excessive amounts of carbon dioxide, carbon monoxide, ammonia and other substances lethal to fish.

There are other stream characteristics which complicate the issue. One problem is that the maximum solubility of oxygen in a stream is relatively low. This concentration is only about 9 ppm of 70°F. As the temperature increases during the warmer months or in thermally polluted areas, the solubility decreases thus yielding the most dangerous stream conditions.

Another stream characteristic that poses a problem is the characteristically small DO range. Most fish need about 5 ppm of DO to maintain life. If the maximum DO level of a stream is 9 ppm, then the range is quite small–especially since the DO concentration can change quite rapidly. Furthermore, normal stream conditions rarely permit the DO to attain its maximum concentration. There is always some pollution in a stream. This pollution may be the residue of some larger amount of pollution introduced upstream. It might also be pollution added in small amounts all along the stream stretch. In any case the dissolved oxygen never attains its maximum concentration.

It is imperative that sufficient oxygen be available in a stream to promote healthy decomposition of organic wastes and healthy fish. The safe dissolved oxygen range is quite small to maintain fish life. Dissolved oxygen levels must be carefully observed since the concentration can drop below desired levels quite rapidly. It is not sufficient to just observe the DO levels since the BOD indicates how much oxygen organic materials will eventually need to complete its irrevocable reactions. Thus one must study both the BOD and DO levels to predict how much pollution is allowed into the stream before the fish habitat is endangered.

## The Objective of This Study

The objective of this study is to use a stochastic approach and predict the biochemical oxygen demand (BOD) and dissolved oxygen (DO) concentrations at several points downstream. In this determination the stream or river stretch will be analyzed as a series of stream segments of homogeneous conditions. Where a major source of pollution is introduced or

where major changes in stream parameters occur, another stream segment should be considered for t days of travel. The approach will be to use generalized initial conditions not available in Thayer and Krutchkoff (6) in the model in the form of probability distributions.

The model used will provide a joint density function for both pollution and dissolved oxygen with given initial conditions. The initial conditions have been generalized to permit the output for one stream segment to be the initial conditions for the succeeding stream segment.

The predicted concentrations are in the form of marginal distributions for both BOD and DO. The conditional distributions are not obtained since their interpretation would be questionable. That is, given a BOD value at some point downstream, we are not interested in the DO distribution. What is of interest is the marginal distributions of BOD and DO along a stream stretch whose parameters may change at known locations.

Included in the appendix is a computer program that assists in the calculations to obtain the marginal distributions. The program utilizes the BOD and DO distributions of one homogeneous stream segment as the initial conditions for the next segment. The program allows for changes in parameter values between one stream segment and the next and continues to calculate new BOD and DO distributions for that segment based on the parameter changes. Aside from predicting the distributions where some of the parameters change values, it also can predict the distributions for any intermediate location along the river. Printing the distributions for locations within a stream segment will not alter or destroy the program's conclusions of that segment provided one desires to print results for successive locations downstream.

# PREVIOUS DEVELOPMENTS

## Deterministic Models

In analyzing a stream's assimilative capacity in terms of pollution and dissolved oxygen, the basis for all models has been the classical theory of Streeter and Phelps (5) in 1925. Very little contributing work has been done until recently. In 1954 Dobbins (2) expanded and generalized the Streeter-Phelps model by including the additional effects of benthal demand, land runoff, and reaeration upon the BOD and DO.

Most all pollution and DO models for a stream stretch make the following three, basic assumptions:

1. The stream flow is considered to be steady and uniform

2. The process along the entire stream stretch is considered to be a steady-state process, that is for any given distance downstream the stream cross-sectional behavior at that point remains unchanged with change in time. This assumption merely implies that the stream geometry and characteristics at a given location are not altered significantly with time.

3. The pollution and dissolved oxygen quantities are uniformly distributed over each cross-section, thus allowing the BOD and DO equations to be described by one-dimensional fluid flow.

With respect to the pollution phase of his model, Dobbins makes three assumptions:

1. Pollution is added at a uniform rate $L_a$ along the route and is measured in parts per million per day (ppm/day). This assumption accounts for minor sources only; a large pollution rate would require separate calculations from that point onward. This rate is commonly known as pollution due to land runoff.

2. Pollution is decreased at the rate of $K_1 L$ due to the action of bacteria, the rate being a first order reaction and proportional to the amount of pollution present. The units of $K_1$ are per day.

3. Pollution is decreased at the rate of $K_3L$ due to sedimentation of solids to the bottom and adsorption of the solids to the rocks and debris. In addition this $K_3L$ term also includes other losses of pollution which do not cause a corresponding loss in dissolved oxygen. The rate of decrease is a first order reaction and proportional to the amount of pollution present. The units are also per day.

Still dealing with pollution, Dobbins uses differential equations to solve for the rate of change of pollution (L) with respect to time of travel (t). Assuming that the stream has a constant velocity, his equation is

$$\frac{dL}{dt} = L_a - (K_1 + K_3)L \quad . \qquad (2.1.1)$$

If $L_A$ ppm is the initial condition of pollution at t= 0, the above equation (2.1.1) has the solution

$$L(t) = L_A e^{-(K_1 + K_3)t} + \frac{L_a}{K_1 + K_3}\left(1 - e^{-(K_1 + K_3)t}\right). \qquad (2.1.2)$$

The L(t) describes the mean value of pollution for a given time of travel.

In dealing with the dissolved oxygen phase of his model, Dobbins makes the following four assumptions:

1. The maximum oxygen solubility concentration under given conditions of temperature and barometric pressure is $C_N'$. This concentration is measured in units of parts per million (ppm).

2. Dissolved oxygen is increased at the rate $K_2(C_N'-C)$ due to reaeration of the stream by the atmospheric oxygen. It is not purely a physical process since it can be adversely affected by solutes in the stream. Its rate is in units of per day.

3. Dissolved oxygen is decreased at the rate of $K_1L$ due to action of bacteria. This parameter $K_1$ is the same $K_1$ discussed previously under the second pollution assumption, and the rate is still

proportional to the amount of pollution present. Therefore, the rate is independent of the concentration of the dissolved oxygen.

4. Dissolved oxygen is decreased at the rate of $D_B$ due to benthal demand, the oxygen used by the bacteria on the bottom of the river. This rate $D_B$ also includes the effects of algae. The rate is independent of the dissolved oxygen present, and it is expressed in units of ppm per day (ppm/day).

The mean value of dissolved oxygen is obtained in a similar manner to that of pollution. Dobbins gives the rate of change of oxygen (C) with respect to time of travel (t) by the equation

$$\frac{dC}{dt} = K_2 (C_N - C) - K_1 L - D_B \quad . \qquad (2.1.3)$$

Note that L appears in this mean value equation of dissolved oxygen where L is the mean value of the previous pollution equation (2.1.1). For an initial condition of $C_o$ at $t = 0$ where L is replaced by equation (2.1.2), the differential equation has the solution

$$C(t) = C_N (1 - e^{-K_2 t}) - \frac{K_1 \left( L_A - \frac{L_a}{K_1 + K_3} \right)}{K_2 - (K_1 + K_3)} \left( e^{-(K_1 + K_3)t} - e^{-K_2 t} \right)$$

$$\left( \frac{D_B}{K_2} + \frac{K_1 L_a}{K_2 (K_1 + K_3)} \right) (1 - e^{-K_2 t}) + C_o \, e^{-K_2 t} \qquad (2.1.4)$$

where $K_2 \neq K_1 + K_3$.

Summarizing, Streeter and Phelps defined the first deterministic model for describing the pollution (BOD) and dissolved oxygen (DO) factors by using differential equations and arriving at values which describe the mean values for both factors. Dobbins expanded the Streeter-Phelps model since he was able to include the effect of the stream processes of land run-off, benthal demand, and reaeration.

In describing the deterministic models and others to follow, one must clearly understand the factor of time. Time or time of travel t occurs in the preceeding differential equations (2.1.1; 2.1.2; 2.1.3; 2.1.4) and relates directly to distance downstream. Naturally one can solve for time by equating

$$d = vt \quad . \tag{2.1.5}$$

The letter d represents the stream distance in units of miles, v represents the average stream velocity in miles per day, and t represents the time of travel in days for the designated stream distance d.

## Stochastic Model

Thayer and Krutchkoff (6) were able to define a Stochastic Model whose means were compatible with Dobbins' mean values for BOD (2.1.2) and DO (2.1.4). In addition Thayer's model yields probability distributions of the pollution and dissolved oxygen concentrations in streams as a function of time of travel. The usefulness of Thayer's probabilistic model is apparent when one recalls that the probabilistic standards were the most realistic that matched the requirements of the stream.

In Thayer and Krutchkoff one assumption was made in addition to those made by Dobbins in the previous section. They proposed to view the problem as a Stochastic Birth and Death Process with BOD and DO being increased and decreased only by small amounts in a very short time interval. Of course the usual assumption of steady and uniform flow, steady state process, and one-dimensional flow are still made.

The method used to find the probability distributions of BOD and DO depend upon the discreteness of the problem and a concept of states. The assumption of a Birth and Death Process allows one to consider that the BOD and DO concentrations increase and decrease in multiples of some small, discrete amount denoted by delta ($\Delta$). (This increase and decrease has been discussed in the form of assumptions in the early part of the section, Deterministic Models). For example consider the assumption that pollution is added at a uniform rate $L_a$ from minor sources along a river stretch. The discreteness of the problem suggests that rather than the pollution being added continuously, the pollution is added by multiples of small pieces of size $\Delta$.

The value of $\Delta$ depends upon the properties of the stream: perhaps turbidity, temperature, and the type of pollution involved. The value of $\Delta$ is estimated by noting that the variance at any given point in the stream is linearly proportional to the distance downstream. One can then use upstream data to estimate $\Delta$.

Related to the discreteness of the problem and the use of $\Delta$ is the concept of states. The term state is defined as follows: A change of size $\Delta$ in the concentration (of pollution or dissolved oxygen) will constitute a change of one state. Thus per state there are $\Delta$ ppm (parts per million) since ppm is the basic unit of measure of BOD and DO and since $\Delta$ is the discrete quantity that, for example, pollution is allowed to increase. The pollution state M will correspond to a concentration such that

$$\text{state M} = (L_M \text{ ppm} \div (\Delta \text{ ppm/state})$$

or simply

$$M = L_M \div \Delta \ . \qquad (2.2.1)$$

The state of dissolved oxygen N also corresponds to a concentration of $C_N$ ppm such that

$$\text{state N} = (C_N \text{ ppm}) \div (\Delta \text{ ppm/state})$$

or simply

$$N = C_N \div \Delta \qquad (2.2.2)$$

The analysis is in terms of probabilities for the possible changes in state in a small increment of time h. The higher order terms containing h will be omitted. It is assumed that a change in more than one state by a factor in time h has probability o(h), which is negligible. The first order probabilities of one change in state and no change in state for the assumptions of increasing and decreasing BOD and DO concentrations (see Deterministic Models) are as follows:

1. The expected amount of increase in BOD due to $L_a$ in small time h is equal to $L_a \cdot h$. (this expression and the following

expectations are the rate assumptions of Dobbins (2) multiplied by time interval h). Let the probability be p (BOD increases by amount $\Delta$ in time interval h due to $L_a$) = $\frac{L_a h}{\Delta}$, and p (no such increase) = $1 - \frac{L_a h}{\Delta}$.

2. The expected decrease in pollution due to sedimentation in time h is $K_3 L_M h$. Then p (BOD increases by amount $\Delta$ in time h due to $K_3$) = $\frac{K_3 L_M h}{\Delta}$, and p (no such decrease) = $1 - \frac{K_3 L_M h}{\Delta}$.

3. The expected decrease in BOD and DO due to bacterial action in time h is $K_1 L_M h$. Then p(BOD and DO decrease by $\Delta$ in h due to $K_1$) = $\frac{K_1 L_M h}{\Delta}$, and p(no such decrease) = $1 - \frac{K_1 L_M h}{\Delta}$.

4. The expected decrease in DO due to benthal demand in time h is $D_B h$. Then p(DO decreases by $\Delta$ in h due to $D_B$) = $\frac{D_B h}{\Delta}$, and p(no such decrease) = $1 - \frac{D_B h}{\Delta}$

5. The expected increase in DO due to reaeration in time h is $K_2(C_N'-C_N)h$. Then p(DO increase by $\Delta$ in h) = $\frac{K_2 (C_N'-C_N) h}{\Delta}$, and p(no such increase) = $1 - \frac{K_2 (C_N' - C_N)}{\Delta}$.

The joint probability of finding the system with pollution in state M and DO in state N at time t is denoted by $p_{M,N}(t)$. Then $p_{M,N}(t+h)$ can be expressed in terms of probabilities at time t in the ways listed below for first order terms. When a factor changes two states or when two or more factors change state, the probability will be o(h).

1. At time t, the BOD is in state M-1, and the DO is on state N. In interval h, the pollution state increases by one and the dissolved oxygen state remains unchanged. The probability of this state occurring is $p_{M-1,N}(t)\left[\frac{L_a h}{\Delta}\right]$.

2. Thus in time h for the states M+1, and N, the associated probability is $p_{M+1,N}(t) \left[\frac{K_3 L_{M+1} h}{\Delta}\right]$

3. Similarly in time h for the states M+1 and N+1, the associated probability is $p_{M+1,N+1}(t) \left[\frac{K_1 L_{M+1} h}{\Delta}\right]$.

4. In addition in time h for the states M and N+1, the associated probability is $p_{M,N+1}(t) \left[\frac{D_B \cdot h}{\Delta}\right]$.

5. Also in time h for the states M and N-1, the associated probability is $p_{M,N-1}(t) \left[\frac{K_2 (C_{N'} - C_{N-1}) h}{\Delta}\right]$.

6. Finally in time h for the states M and N, the associated probability is $p_{M,N}(t) \left[1 - \frac{L_a h}{\Delta} - \frac{K_3 L_M h}{\Delta} - \frac{K_1 L_M h}{\Delta} - \frac{D_B h}{\Delta} - \frac{K_2 (C_{N'} - C_N) h}{\Delta}\right]$

The coefficients of all other possible probabilities are o(h).

Combining the disjoint probabilities one obtains the following difference equation:

$$p_{M,N}(t+h) = \left[1 - \frac{L_a h}{\Delta} - K_3 Mh - K_1 Mh - \frac{D_B h}{\Delta} - K_2 \cdot \left(\frac{C_{N'}}{\Delta} - N\right)h\right] \cdot p_{M,N}(t) + \frac{L_a h}{\Delta} \cdot p_{M-1,N}(t) + K_3 \cdot (M+1)h \cdot p_{M+1,N}(t)$$

$$+ K_1 (M+1)h \cdot p_{M+1,N+1}(t) + \frac{D_B \cdot h}{\Delta} \cdot p_{M,N+1}(t)$$

$$+ K_2 \frac{C_{N'}}{\Delta} - (N-1)h \cdot p_{M,N-1}(t) \quad . \qquad (2.2.3)$$

By subtracting $p_{M,N}(t)$ from both sides of the above equation (since t is continuous), dividing both sides by h, and taking the limit as h approaches zero, one obtains a differential difference equation which ultimately yields the probability distributions for both BOD and DO.

A standard method is then used, namely probability generating functions (or factorial generating functions), to solve the differential difference equation mentioned above. The problem was then solved for the following three initial conditions:

1. There is a point source of pollution and dissolved oxygen.

2. There is no major source of pollution other than pollution due to land run-off.

3. The pollution and dissolved oxygen states have independent binomial distributions whose means and variances can be measured.

Although the Stochastic equation (2.2.3) was solved for all three of the above initial conditions, only the first initial condition will be of interest in this paper. The solutions based on the initial point sources of BOD and DO parallels the derivations in the chapter, Theoretical Results of the Stochastic Model.

Considering the initial condition of a point source of pollution and dissolved oxygen, Thayer and Krutchkoff found the marginal distributions, means, and variances of both BOD and DO for some time of travel t.

The probability that pollution is in state M at time t is given as

$$p_M(t) = \exp\left[\frac{-L_a}{\Delta(K_1 + K_3)}\,(1 - e^{-(K_1 + K_3)\,t})\right] \cdot \sum_{i=0}^{M^*} \binom{\frac{L_A}{\Delta}}{i}$$

$$\cdot\,(1 - e^{-(K_1 + K_3)t})^{\frac{L_A}{\Delta} i}\,(e^{-(K_1 + K_3)t})^{i}$$

$$\frac{\left[ \frac{L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) \right] M-i}{(M-i)!} \qquad (2.2.4)$$

where

$$M^* = \text{minimum} \left\{ L_A/\Delta; M \right\}$$

For the mean amount of pollution present, they obtained

$$L(t) = L_A e^{-(K_1 + K_3)t} + \frac{L_a}{K_1 + K_3} (1-e^{-(K_1 + K_3)t}) \qquad (2.2.5)$$

and for the variance of the amount of pollution,

$$\text{BOD variance} = \Delta \left[ \frac{L_a}{K_1 + K_3} + L_A e^{-(K_1 + K_3)t} \right] \left[ 1-e^{-(K_1 + K_3)t} \right]$$

(2.2.6)

The mean amount of pollution is equivalent to Dobbins' equation (2.1.2) as was expected.

The dissolved oxygen was treated in the same manner. The probability that DO is in state N at time of travel t is

$$p_N(t) = \exp \left[ \frac{-L_a K_1}{\Delta(K_1 + K_3)|K_2 - (K_1 + K_3)|} (1-e^{-(K_1 + K_3)t}) \right.$$

$$\left. + \frac{L_a K_1}{\Delta K_2 |K_2 - (K_1 + K_3)|} (1-e^{-K_2 t}) - \frac{D_B}{\Delta K_2} (1-e^{-K_2 t}) \right]$$

$$\cdot \sum_{i=0}^{N^*} \sum_{j=0}^{N^{**}} \binom{L_A/\Delta}{i} \left[1 - \frac{K_1}{K_2 - (K_1 + K_3)} \left(e^{-(K_1 + K_3)t} - e^{-K_2 t}\right)\right]^{L_A/\Delta - i}$$

$$\cdot \left[\frac{K_1}{K_2 - (K_1 + K_3)} \left(e^{-(K_1 + K_3)t} - e^{K_2 t}\right)\right]^{i} \binom{\frac{C_{N'} - C_o}{\Delta}}{j}$$

$$\cdot (1 - e^{-K_2 t})^{\frac{C_{N'} - C_o}{\Delta} - j} (e^{-K_2 t})^{j} \cdot \left[\frac{L_a K_1}{\Delta (K_1 + K_3)\left[K_2 - (K_1 + K_3)\right]}\right]$$

$$\cdot (1 - e^{-K_1 + K_3)t}) - \frac{L_a K_1}{\Delta K_2 \left[K_2 - (K_1 + K_3)\right]} (1 - e^{-K_2 t})$$

$$+ \frac{D_B}{\Delta K_2} (1 - e^{-K_2 t}) \Bigg]^{\frac{C_{N'}}{\Delta} - N - j - i} \cdot \frac{1}{\left(\frac{C_{N'}}{\Delta} - N - j - i\right)!} \qquad (2.2.7)$$

for

$$K_2 \neq (K_1 + K_3)$$

where

$$N^* = \text{minimum} \left\{L_A/\Delta \; ; \; \frac{C_{N'}}{\Delta} - N\right\}$$

and

$$N^{**} = \text{minimum} \left\{\frac{C_{N'} - C_o}{\Delta} \; ; \quad \frac{C_{N'}}{\Delta} - N - i\right\}$$

The mean amount of dissolved oxygen is

$$C(t) = C_{N'} - (C_{N'} - C_o)\, e^{-K_2 t} - \frac{D_B}{K_2}\,(1-e^{-K_2 t})$$

$$- \frac{K_1 L_a}{(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]}\,(1-e^{-(K_1 + K_3)t}) + \frac{L_a K_1}{K_2\,[K_2 - (K_1 + K_3)\,]}$$

$$\cdot\,(1-e^{-K_2 t}) - L_A\,\frac{K_1}{K_2 - (K_1 + K_3)}\,(e^{-(K_1 + K_3)t} - e^{-K_2 t})\,, \qquad (2.2.8)$$

and the variance of the amount of dissolved oxygen is

$$\text{DO Variance} = \Delta \left\{ (C_{N'} - C_o)\,(1-e^{-K_2 t})\, e^{-K_2 t} + \frac{D_B}{K_2}\,(1-e^{-K_2 t}) \right.$$

$$+ \frac{L_a K_1}{(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]}\,(1-e^{-(K_1 + K_3)t})$$

$$- \frac{L_a K_1}{K_2\,[K_2 - (K_1 + K_3)\,]}\,(1 - e^{-K_2 t}$$

$$+ L_A\,\frac{K_1}{K_2 - (K_1 + K_3)}\,(e^{-(K_1 + K_3)t} - e^{-K_2 t})\,\Big[1$$

$$\left. - \frac{K_1}{K_2 - (K_1 + K_3)}\,\cdot\,(e^{-K_1 + K_3)t} - e^{-K_2 t})\Big]\right\} \qquad . \qquad (2.2.9)$$

where for both equations (2.2.8; 2.2.9)

$$K_2 \neq (K_1 + K_3) \quad .$$

## THEORETICAL RESULTS OF THE STOCHASTIC MODEL

### Stochastic Solution

In order to solve the differential difference equation (2.2.3) in the preceeding section, the general method of utilizing probability generating functions was used. To simplify mathematics, the three steps were rearranged from their usual sequence.

The probability generating function (abbreviated as generating function) is defined as

$$P(S,R,t) = \sum_{M=O}^{\infty} \sum_{N=O}^{\infty} S^M R^N \, p_{M,N}(t)$$

where $p_{M,N}(t)$ is the joint probability that the pollution is in state M and the dissolved oxygen is in state N.

The first step involves a relationship of $A = \psi(B,C)$, which has already been derived by Thayer and Krutchkoff as

$$A = \ln P \left[1-B- \frac{K_1}{K_2 - (K_1 + K_3)} \cdot \frac{1}{C} , \frac{C}{1+C} ; O\right]$$

$$+ \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} \left(\frac{1+C}{C}\right) + \frac{L_a B}{\Delta(K_1 + K_3)} - \frac{D_B}{\Delta K_2} \left(\frac{1+C}{C}\right)$$

$$- \frac{C_{N'}}{\Delta} \ln \left(\frac{C}{1+C}\right) . \qquad (3.1.1)$$

Note that the first term of this equation is a probability generating function P(S,R;t) where

$$S = 1 - B - \frac{K_1}{K_2 - (K_1 + K_3)} \qquad (3.1.2)$$

$$R = \frac{C}{1+C} \tag{3.1.3}$$

and

$$t = 0 \tag{3.1.4}$$

The first step is to substitute the initial generating function P(S,R;t) into equation (3.1.1).

The second step is to replace A, B, and C in equation (3.1.1) by their respective equals, that is

$$A = 1n\, P(S,R;t) + \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} \cdot \frac{1}{R} \quad \frac{L_a}{\Delta(K_1 + K_3)} \cdot (1\text{-}S)$$

$$- \frac{L_a K_1}{\Delta(K_1 + K_3)[K_2 - (K_1 + K_3)]} \cdot \left(\frac{1\text{-}R}{R}\right) - \frac{D_B}{\Delta K_2} \cdot \frac{1}{R} - \frac{C_{N'}}{\Delta} \quad 1n\, R \tag{3.1.5}$$

$$B = \frac{-K_1}{K_2 - (K_1 + K_3)} \left(\frac{1\text{-}R}{R}\right) e^{-(K_1 + K_3)t} + (1\text{-}S)\, e^{-(K_1 + K_3)t} \tag{3.1.6}$$

and

$$C = \frac{R}{1\text{-}R} e^{-K_2 t} \tag{3.1.7}$$

The third step is to simplify the expression and solve for P(S,R;t).

## Generalized Initial Conditions

Although Thayer and Krutchkoff discussed only three types of initial conditions (as mentioned in Stochastic Model), these conditions can be generalized. At time of travel t = 0 for some stream location, the states of pollution and dissolved oxygen can both have probability distributions. These initial distributions will include the three types of initial conditions as special cases. If one wishes to consider an initial point source of BOD and DO, then the probability of that particular state is one, while all other states have zero probability. If considering an essentially clean stream, the initial probability distributions will have large variances and small means. The fact that the probabilities of all the states can be specified as an initial condition precludes all discussion as to postulating the pollution and dissolved oxygen distributions as Thayer and Krutchkoff did.

The object of this section is to use the generalized initial conditions and derive the marginal probabilities for both pollution and DO being in particular states for some time of travel t. In so doing, the three steps mentioned in the previous section will be instrumental in obtaining the necessary probability generating functions. In turn the generating functions will ultimately yield the desired probabilities.

At t = 0, the BOD is in state $L_A/\Delta$, and the DO is in state $C_o/\Delta$ with a certain joint probability. Using generating functions, these initial conditions can be expressed as

$$P(S,R;0) = \sum_{\frac{L_A}{\Delta}=0}^{\infty} \sum_{\frac{C_o}{\Delta}=0}^{\infty} S^{\frac{L_A}{\Delta}} R^{\frac{C_o}{\Delta}} P_{\frac{L_A}{\Delta},\frac{C_o}{\Delta}}(0) \quad (3.2.1)$$

or

$$P(S,R;0) = \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} S^L R^D P_{L,D}(0) \quad (3.2.2)$$

if $L = \frac{L_A}{\Delta}$ and $D = \frac{C_o}{\Delta}$ for simplification.

As mentioned in Step 1, S, R, and t are substituted for their respective equals (3.1.2; 3.1.3; 3.1.4) into the initial generating function resulting in

$$P\left(1\text{-}B - \frac{K_1}{K_2 - (K_1 + K_3)} \cdot \frac{1}{C} \; ; \; \frac{C}{1+C} \; ; 0\right) = \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} \left[ 1\text{-}B \right.$$

$$\left. - \frac{K_1}{K_2 - (K_1 + K_3)} \cdot \frac{1}{C} \right]^{L} \left[ \frac{C}{1+C} \right]^{D} p_{L,D}(0) \cdot \qquad (3.2.3)$$

Substituting this result into equation (3.1.1), one obtains

$$A = 1n \left\{ \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} \left[ 1\text{-}B - \frac{K_1}{K_2 - (K_1 + K_3)} \cdot \frac{1}{C} \right]^{L} \left[ \frac{C}{1+C} \right]^{D} p_{L,D}(0) \right\}$$

$$+ \left\{ \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} \left( \frac{1+C}{C} \right) + \frac{L_a}{\Delta (K_1 + K_3)} \cdot B - \frac{D_B}{\Delta K_2} \left( \frac{1+C}{C} \right) \right.$$

$$\left. + 1n \left( \frac{C}{1+C} \right)^{-\frac{C_{N'}}{\Delta}} \right\} \qquad (3.2.4)$$

Replace A, B, and C by their equals (3.1.5; 3.1.6; 3.1.7) into the equation immediately above, as step 2 required. Then one can raise all factors to the Naperian base (e) to obtain

$$P(S,R;t) \cdot \frac{C_{N'}}{\Delta} \cdot \exp \left\{ \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} \cdot \frac{1}{R} \right.$$

$$+ \frac{L_a}{\Delta(K_1 + K_3)} (1-S) - \frac{L_a K_1}{\Delta(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]} \left(\frac{1-R}{R}\right)$$

$$- \frac{D_B}{\Delta K_2} \cdot \frac{1}{R} = \left\{ \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} \left[ 1 + \frac{K_1}{K_2 - (K_1 + K_3)} \left(\frac{1-R}{R}\right) \right.\right.$$

$$\cdot e^{-(K_1 + K_3)t} - (1-S)\, e^{-(K_1 + K_3)t} - \frac{K_1}{K_2 - (K_1 + K_3)} \left(\frac{1-R}{R}\right) e^{-K_2 t} \left.\right]^{L}$$

$$\cdot \left[ \frac{\frac{Re}{1-R}^{-K_2 t}}{1 + \frac{Re}{1-R}^{-K_2 t}} \right]^{D} P_{L,D}\,(0) \left.\right\} \cdot \left[ \frac{\frac{Re}{1-R}^{-K_2 t}}{1 + \frac{Re}{1-R}^{-K_2 t}} \right]^{\frac{C_{N'}}{\Delta}}$$

$$\cdot \exp \left\{ \frac{L_a K_1}{\Delta K_2\,[K_2 - (K_1 + K_3)\,]} \cdot \frac{1 + \frac{Re}{1-R}^{-K_2 t}}{\frac{Re}{1-R}^{-K_2 t}} \right.$$

$$- \frac{L_a K_1}{\Delta(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]} \cdot \left(\frac{1-R}{R}\right) \cdot e^{-(K_1 + K_3)t}$$

$$\left. + \frac{L_a}{\Delta(K_1 + K_3)} (1-S)\, e^{-(K_1 + K_3)t} - \frac{D_B}{\Delta K_2} \cdot \left[ \frac{1 + \frac{Re}{1-R}^{-K_2 t}}{\frac{Re}{1-R}^{-K_2 t}} \right] \right\}$$

(3.2.5)

Simplifying and solving for P(S,R;t) as stated in step 3, one obtains

$$P(S,R;t) = \exp\left\{ \frac{L_a K_1}{\Delta(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]} \left(\frac{1-R}{R}\right) (1 - e^{-(K_1 + K_3)\,t}) \right.$$

$$- \frac{L_a K_1}{\Delta K_2\,[K_2 - (K_1 + K_3)\,]} \left(\frac{1-R}{R}\right) (1 - e^{-K_2 t}) - \frac{L_a}{\Delta(K_1 + K_3)} (1-S)\,(1 - e^{-(K_1 + K_3)\,t})$$

$$\left. + \frac{D_B}{\Delta K_2} \left(\frac{1-R}{R}\right) (1 - e^{-K_2 t}) \right\} \cdot \left[ 1 + \left(\frac{1-R}{R}\right) e^{-K_2 t} \right]^{\frac{C_N'}{\Delta} \cdot R \frac{C_N'}{\Delta}}$$

$$\cdot \left\{ \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} \left[ 1 + \frac{K_1}{K_2 - (K_1 + K_3)} \left(\frac{1-R}{R}\right) (e^{-(K_1 + K_3)\,t} - e^{-K_2 t}) \right.\right.$$

$$\left.\left. - (1-S)\, e^{-(K_1 + K_3)\,t} \right]^{L} \cdot \left[ 1 + \left(\frac{1-R}{R}\right) e^{-K_2 t} \right]^{-D} P_{L,D}(0) \right\} . \qquad (3.2.6)$$

This equation is the joint probability generating function for both pollution and dissolved oxygen. This function can be expanded in a power series. The coefficient of $S^M R^N$ is the quantity $p_{M,N}(t)$ which is the probability that pollution is in state M while dissolved oxygen is in state N.

To derive the marginal probabilities of both pollution and DO, it is best to first obtain the marginal generating functions. The marginals of a bivariate generating function are obtained by equating the unwanted variable to the number one.

Thus the marginal generating function for pollution is obtained by equating R = 1 in (3.2.6). Then we have

$$P(S,1;t) = \exp\left\{ \frac{-L_a}{\Delta(K_1 + K_3)} (1-S)(1-e^{-(K_1 + K_3)t}) \right\} \cdot \left\{ \sum_{L=0}^{\infty} \left[ 1 \right. \right.$$

$$\left. \left. - (1-S)\, e^{-(K_1 + K_3)t} \right]^L \; p_L(0) \right\} \qquad (3.2.7)$$

$$= \exp\left\{ \frac{L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) \right\} \cdot \exp\left\{ \frac{L_a}{\Delta(K_1 + K_3)} \right.$$

$$\left. \cdot (1-e^{-(K_1 + K_3)t})\, S \right\} \cdot \left\{ \sum_{L=0}^{\infty} \left[ 1-e^{-(K_1 + K_3)t} \right) \right.$$

$$\left. + S\, e^{-(K_1 + K_3)t} \right]^L p_L(0) \right\} \qquad (3.2.8)$$

merely by rearranging terms.

It is important to realize that the definition of marginal distributions was used in the above derivation. After equating R = 1 in equation (3.2.6) part of that joint generating function can be reduced as

$$\sum_{L=0}^{\infty} \sum_{D=0}^{\infty} f(L)\, p_{L,D}(0) = \sum_{L=0}^{\infty} f(L)\, p_L(0) \qquad (3.2.8A)$$

for some function f(L). No assumption of independence has been made to reduce the joint probabilities to a marginal.

In order to find the coefficient of $S^M$ the above marginal generating function (3.2.8) can be expanded in a power series in S. The result will be the probability that the pollution is in state M at time t. Let

$$F = \frac{L_a}{\Delta(K_1 + K_3)}$$

and

$$G = e^{-(K_1 + K_3)t}$$

Then

$$K = \exp\left\{-F(1-G)\right\}$$

which is a constant since it does not contain an S. By substitution the marginal (3.2.8) can be rewritten as

$$P(S,1;t) = K \cdot \exp\left\{F(1-G)S\right\} \cdot \left\{\sum_{L=0}^{\infty} [(1-G) + GS]^L p_L(0)\right\} \cdot \tag{3.2.9}$$

In addition, let

$$a_i = \frac{\left\{F(1-G)\right\}^i}{i!}$$

for the coefficients of an exponential series expansion, and

$$b_{L,j} = \binom{L}{j} (1-G)^{L-j} G^{j} \cdot p_L(0)$$

for the coefficients of a binomial series expansion. Rewriting the above marginal (3.2.9) with the appropriate exponential and binomial series expansions coefficients, one obtains

$$P(S,1;t) = K \cdot (a_o + a_1 S^1 + a_2 S^2 + \dots) \cdot \left\{ \sum_{L=0}^{\infty} (b_{L,o} + b_{L,1} S^1 + \dots + b_{L,L} S^L) \right\}$$

Expanding the terms of the binomial coefficients, one acquires the form

$$P(S,1,t) = K \cdot (a_o + a_1 S^1 + \dots)[b_{oo} + (b_{1o} + b_{11} S^1) + b_{20} + b_{21} S^1 + b_{22} S^2) + (b_{30} + b_{31} S^1 + b_{32} S^2 + b_{33} S^3) + \dots]$$

(3.2.10)

In practice the number of binomial terms available is equal to the number of available $p_L(0)$ since $b_{L,j}$ contains a $p_L(0)$ term. If one is interested in the probability of the zero state of pollution, that is where M = 0, then in collecting coefficients of powers of $S^0$ one obtains

$$p_{M=0}(t) = K(a_o b_{oo} + a_o b_{10} + a_o b_{20} + \dots)$$

$$= K \cdot a_o \sum_{K=0}^{\infty} b_{K,0} \quad .$$

The probability of the first state of pollution (M=1) is

$$P_{M=1}(t) = K[(a_1 b_{oo} + a_1 b_{10} + a_1 b_{20} + \dots)$$

$$+ (a_o b_{11} + a_o b_{21} + \dots)]$$

$$= K \cdot a_1 \sum_{K=0}^{\infty} b_{K,0} + K \cdot a_o \sum_{K=1}^{\infty} b_{K,1}$$

and similarly for the second state (M=2)

$$P_{M=2}(t) = K \cdot a_2 \sum_{K=0}^{\infty} b_{K,0} + K \cdot a_1 \sum_{K=1}^{\infty} b_{K,1}$$

$$+ K \cdot a_o \sum_{K=2}^{\infty} b_{K,2} \cdot$$

In general the probability of the $M^{th}$ state of pollution is

$$P_M(t) = K \cdot \sum_{i=0}^{M^*} \sum_{K=i}^{\infty} a_{M-i}\, b_{K,i}$$

or, with all substitutions made,

$$P_M(t) = \exp\left\{\frac{-L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t})\right\} \cdot \left\{\sum_{i=0}^{M^*} \sum_{L=i}^{\infty} \binom{L}{i}\right.$$

$$\cdot (1-e^{-(K_1 + K_3) t_j})^{L} (e^{-(K_1 + K_3) t_j})^{i} \; p_L(0)$$

$$\frac{\left[\frac{L_a}{\Delta (K_1 + K_3)} \; (1-e^{-(K_1 + K_3) t_j}\right]^{M-i}}{(M-i)!} \qquad (3.2.11)$$

where $M^*$ = minimum [M;L] . In practice all entries in the above expression (3.2.11) will be known, including the initial marginal probabilities $p_L(0)$ for the pollution state M.

Note the similarity in the probability of the $M^{th}$ pollution state using general initial conditions (3.2.11) to the results of Thayer and Krutchkoff using initial conditions of a point source of pollution (2.2.4). The difference in the two is the fact that in the above result, one sums over all initial pollution states whereas Thayer and Krutchkoff only used a single initial state.

The dissolved oxygen equations are quite similar to the pollution equations in derivation except that an assumption of independence will be made for the first time in this paper. The assumption is that pollution and dissolved oxygen states are statistically independent of each other. Recall that in the previous discussion of the marginal probability generating function for pollution, this assumption was not necessary (equation (3.2.9)). The assumption is made realizing that strict statistical independence is not the case, but that the assumption allows the derivation to continue without unruly mathematical complications. A study of independence will follow in the chapter, Study of Independence, revealing that the assumption does not appear to be detrimental to the results.

The dissolved oxygen probability will also be derived from the joint probability generating function (3.2.6) by equating S=1 and assuming independence. Doing this one obtains

$$p(1,R;t) = \exp \left\{ \frac{L_a K_1}{\Delta (K_1 + K_3) \; [K_2 - (K_1 + K_3)]} \; \left(\frac{1-R}{R}\right) \; (1-e^{-(K_1 + K_3) t_j}\right.$$

$$- \frac{-L_a K_1}{\Delta K_2 K_2 - (K_1 + K_3)} \left(\frac{1-R}{R}\right) (1-e^{-K_2 t}) + \frac{D_B}{\Delta K_2} \left(\frac{1-R}{R}\right) (1-e^{-K_2 t}) \Bigg\}$$

$$\cdot (1- \left(\frac{1-R}{R}\right) e^{-K_2 t}) \frac{C_{N'}}{\Delta} \cdot R \frac{C_{N'}}{\Delta} \cdot \Bigg\{ \sum_{L=0}^{\infty} \sum_{D=0}^{\infty} [1$$

$$+ \frac{K_1}{K_2 - (K_1 + K_3)} \left(\frac{1-R}{R}\right) (e^{-(K_1 + K_3) t} - e^{-K_2 t})]^L \cdot P_L(0)$$

$$\cdot [1 + \left(\frac{1-R}{R}\right) e^{-K_2 t}]^{-D} \cdot P_{D'}(0) \Bigg\} \cdot \qquad (3.2.12)$$

The power series expansion of this generating function can be obtained in a manner similar to the case of pollution. The coefficient of the $N^{th}$ power of S, hence the desired probability of the $N^{th}$ state of DO, is

$$P_N(t) = \exp \Bigg\{ \frac{-L_a K_1}{\Delta(K_1 + K_3)[K_2 - (K_1 + K_3)]} (1-e^{-(K_1 + K_3) t})$$

$$+ \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} (1-e^{-K_2 t})$$

$$- \frac{D_B}{\Delta K_2} (1-e^{-K_2 t}) \Bigg\} \cdot \sum_{i=0}^{N^*} \sum_{L=i}^{\infty} \binom{L}{i} (1- \frac{K_1}{K_2 - (K_1 + K_3)}$$

$$\cdot (e^{-(K_1 + K_3) t} - e^{-K_2 t})^{L-i} \left( \frac{K_1}{K_2 - (K_1 + K_3)} (e^{-(K_1 + K_3) t} - e^{-K_2 t}) \right)^i$$

$$\cdot p_L(0) \cdot \sum_{j=0}^{N^{**}} \sum_{D=j}^{\frac{C_{N'}}{\Delta}} \binom{\frac{C_{N'}}{\Delta} - D}{j} (1-e^{-K_2 t})^{\frac{C_{N'}}{\Delta} - D - j}$$

$$\cdot (e^{-K_2 t})^j \cdot \left[ \frac{L_a K_1}{\Delta(K_1 + K_3)[K_2 - (K_1 + K_3)]} (1-e^{-(K_1 + K_3) t} \right.$$

$$- \frac{L_a K_1}{\Delta(K_1 + K_3)[K_2 - (K_1 + K_3)]} (1-e^{-K_2 t})$$

$$\left. + \frac{D_B}{\Delta K_2} (1-e^{-K_2 t}) \right]^{\frac{C_{N'}}{\Delta} - N-j-i} \cdot \frac{1}{(\frac{C_{N'}}{\Delta} - N-j-i)!} \cdot p_D(0) \qquad (3.2.13)$$

where

$$N^* = \min \left\{ \frac{C_{N'}}{\Delta} - N \; ; \; L \right\}$$

and

$$N^{**} = \min\left\{\frac{C_{N'}}{\Delta} - N - i\,;\ \frac{C_{N'}}{\Delta} - D\right\}$$

The power series expansion in powers of R has exponents of R ranging from

to - ∞. The coefficient of $R^N$, specifically $p_N(t)$, is the probability of being in state N at time t. The negative exponents occurred when a term was dropped (not previously discussed) from the solution of the partial differential equation. However, the coefficients when N is negative are so small that they can be neglected. Hence only the powers of R in the range from to zero need to be of concern.

## Means and Variances

The method employed here to find means and variances for pollution and dissolved oxygen will be that of probability generating functions (or factorial moments). More specifically, the mean amount of pollution and dissolved oxygen is found by first solving for the mean state and then by multiplying by a simple factor. The variance of the amount of BOD and DO is found by first solving for the variance in states and then making a similar, simple multiplication.

The mean state of pollution is denoted as ES and is obtained from the appropriate marginal factorial moment (3.2.8). Taking the first partial derivative with respect to S and evaluating at S=1, one acquired the following mean:

$$\frac{\partial P(S,1;t)}{\partial S} = \frac{\partial}{\partial S}\left\{\exp\left[\frac{-L_a K_1}{\Delta(K_1 + K_3)}\left(1-e^{-(K_1 + K_3)t}\right)(1-S)\right]\right.$$

$$\left.\cdot \sum_{L=0}^{\infty}\left[1-e^{-(K_1 + K_3)t(1-S)}\right]^L p_L(0)\right\}$$

$$= \exp\left[\frac{-L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) (1-S)\right] \cdot$$

$$\cdot \sum_{L=0}^{\infty} \Big[ 1-e^{-(K_1 + K_3)t}$$

$$\cdot(1-S)\Big]^{L-1} (L \cdot e^{-(K_1 + K_3)t}) p_L(0) + \Big\{ \sum_{L=0}^{\infty} [1$$

$$-e^{-(K_1 + K_3)t}(1-S)]^{L} p_L(0) \cdot \frac{L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t})$$

$$\cdot \exp\left[\frac{-L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) (1-S)\right] \Big\}$$

$$\left.\frac{\partial P(S,1;t)}{\partial S}\right|_{S=1} = e^{-(K_1 + K_3)t} \cdot \sum_{L=0}^{\infty} L \cdot p_L(0)$$

$$+ \frac{L_a}{\Delta(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) \cdot \qquad (3.3.1)$$

Note that the above mean pollution state is still dependent upon delta ($\Delta$) in both terms; the second term has a $\Delta$ which is visible; and the second term includes a $\Delta$ since the section, Generalized Initial Conditions, defined for simplicity.

Multiplying the mean pollution state by $\Delta$, one obtains the mean AMOUNT of pollution which is not dependent upon $\Delta$ as a function of time:

$$\text{Pollution mean } (t) = \frac{L_a}{(K_1 + K_3)} (1-e^{-(K_1 + K_3)t}) + e^{-(K_1 + K_3)t} \cdot \Delta$$

$$\cdot \sum_{L=0}^{\infty} L \cdot p_L(0) \cdot \qquad (3.3.2)$$

The significance of this result together with the variance of pollution will be discussed later in this section.

The variance is easily obtained from the second partial derivative ES(S-1). The expression is

$$\text{Variance } S = [ES(S-1) + ES] - (ES)^2 \qquad (3.3.3)$$

where

$$ES(S-1) = \left. \frac{\partial^2 P(S,1;t)}{\partial S^2} \right|_{S=1}$$

and

$$ES = \left. \frac{\partial P(S,1;t)}{\partial S} \right|_{S=1}$$

Using the same marginal generating function, one evaluates the above equation (3.3.3) to yield the variance of pollution states. Multiplying by $\Delta^2$ yields an expression for the variance in AMOUNT of pollution as a function of time:

$$\text{Pollution Variance } (t) = \Delta \left[ \frac{L_a}{(K_1 + K_3)} + e^{-(K_1 + K_3)t} \cdot \Delta \cdot \sum_{L=0}^{\infty} L \cdot p_L(0) \right]$$

$$\cdot\ [\ 1\text{-}e^{-(K_1 + K_3)\,t}]\ . \qquad (3.3.4)$$

In solving for the mean and variance of dissolved oxygen, the same procedure is followed as in the case of pollution. The mean state of dissolved oxygen is found by taking the first partial derivative of the appropriate marginal probability generating function (3.2.12), evaluating the partial derivative at R = 1, and assuming independence of BOD and DO. Solving, one acquires the following:

$$\frac{\partial P\,(1,R;t)}{\partial R} = \frac{\partial}{\partial R}\left\{ \exp \frac{L_a K_1}{\Delta(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]}\,(1\text{-}e^{-(K_1 + K_3)\,t})\,(\frac{1\text{-}R}{R})\right.$$

$$- \frac{L_a K_1}{\Delta K_2\,[\,K_2\text{-}(K_1 + K_3)\,]}\,(1\text{-}e^{-K_2 t})(\frac{1\text{-}R}{R}) + \frac{R}{\Delta K_2}\,(1\text{-}e^{-K_2\,t})(\frac{1\text{-}R}{R})$$

$$\cdot\left[\ 1 + e^{-K_2\,t}\,(\frac{1\text{-}R}{R})\right]^{\frac{C_{N'}}{\Delta}} \cdot R^{\frac{C_{N'}}{\Delta}} \cdot \sum_{L=0}^{\infty} \left[1\right.$$

$$+ \frac{K_1}{K_2 - (K_1 + K_3)}\,(e^{-(K_1 + K_3)\,t} - e^{-K_2\,t})\,(\frac{1\text{-}R}{R})\ \Big]^{L} P_L\,(0)$$

$$\cdot \sum_{D=0}^{\infty} \left[1 + e^{-K_2\,t}\,(\frac{1\text{-}R}{R})\right]^{-D} P_D\,(0)\Big\}$$

and

$$\left.\frac{\partial P(1,R;t)}{\partial R}\right|_{R=1} = \frac{C_{N'}}{\Delta} + e^{-K_2 t} \cdot \sum_{D=0}^{\infty} D \cdot p_D(0) - \frac{C_{N'}}{\Delta} e^{-K_2 t}$$

$$- \frac{K_1}{K_2 - (K_1 + K_3)} (e^{-(K_1 + K_3) t} - e^{-K_2 t}) \cdot \sum_{L=0}^{\infty} L \cdot p_L(0)$$

$$- \frac{L_a K_1}{\Delta(K_1 + K_3)[K_2 - (K_1 + K_3)]} (1 - e^{-(K_1 + K_3) t})$$

$$+ \frac{L_a K_1}{\Delta K_2 [K_2 - (K_1 + K_3)]} \cdot (1 - e^{-K_2 t}) - \frac{D_B}{\Delta K_2} (1 - e^{-K_2 t}) . \qquad (3.3.5)$$

Multiplying the above mean state equation by $\Delta$ yields the mean AMOUNT of dissolved oxygen as a function of time:

$$\text{DO mean}(t) = C_{N'} + e^{-K_2 t} \cdot \sum_{D=0}^{\infty} D \cdot p_D(0) - C_{N'} e^{-K_2 t}$$

$$\frac{K_1}{K_2 - (K_1 + K_3)} (e^{-(K_1 + K_3) t} - e^{-K_2 t}) \cdot \sum_{L=0}^{\infty} L \cdot p_L(0)$$

$$- \frac{L_a K_1}{\Delta(K_1 + K_3)\,[K_2 - (K_1 + K_3)\,]}\ (1-e^{-(K_1 + K_3)\,t})$$

$$+ \frac{L_a K_1}{\Delta K_2\,[K_2 - (K_1 + K_3)\,]} \cdot (1-e^{-K_2 t}) - \frac{D_B}{\Delta K_2}\ (1-e^{-K_2 t}) \quad .$$

(3.3.6)

Following the same technique as the pollution case, one obtains the variance of the dissolved oxygen state by taking the second partial derivative of the appropriate marginal, evaluating at R=1, adding on the mean as a correction factor, and then subtracting the square of the mean. Multiplying by $\Delta^2$, one obtains the variance of the AMOUNT of dissolved oxygen:

$$\text{DO variance}\ (t) = \Delta \left\{ \frac{L_a K_1}{(K_1 + K_3)\ [K_2 - (K_1 + K_3)\,]}\ (1-e^{-(K_1 + K_3)\,t}) \right.$$

$$- \frac{L_a K_1}{K_2\,[K_2 - (K_1 + K_3)\,]}\ (1-e^{-K_2 t}) + \frac{D_B}{\Delta K_2}\ (1-e^{-K_2 t})$$

$$+ \frac{C_{N'}}{\Delta}\ (1-e^{-K_2 t})\ e^{-K_2 t} - (1-e^{-K_2 t})\, e^{-K_2 t}$$

$$\cdot \sum_{D=0}^{\infty} D \cdot p_D(0) + \frac{K_1}{K_2 - (K_1 + K_3)}\ (e^{-(K_1 + K_3)\,t} - e^{-K_2 t})$$

$$\cdot \left[1 - \frac{K_1}{K_2 - (K_1 + K_3)} \cdot (e^{-(K_1 + K_3)t} - e^{-K_2 t}) \right.$$

$$\left. \cdot \sum_{L=0}^{\infty} L \cdot p_L(0) \right] \Bigg\} \quad . \qquad (3.3.7)$$

Consider the mean pollution equation (3.3.2), the mean dissolved oxygen equation (3.3.6), the BOD variance equation (3.3.4), and the DO variance equation (3.3.7). The results mentioned above are the same as previous results for point source initial conditions except that the above equations have a distribution of pollution values ($L=\frac{L_A}{\Delta}$) with their probabilities and a distribution of DO values ($D=\frac{C_e}{\Delta}$) with their assigned probabilities. (In the case of mean pollution equation, the DO distribution is degenerate). Under the special conditions of a point source of pollution ($\frac{L_A}{\Delta}$) with probability equal to one and a point source of DO ($\frac{C_o}{\Delta}$) with probability one, all four above means and variances correspond to the previous results (2.2.5; 2.2.8; 2.2.6; 2.2.9).

## Computer Program Discussion

In order to reduce the amount of calculations to a minimum and to make the results of this theory more meaningful, a computer program was written to handle the tedious work. The program is based on a stream which is measured in length by time of travel. This time can be readily converted into distance by the formula given by (2.1.5). A stream is divided into stream segments that are homogeneous in the parameters; a segment also immediately follows a point source of pollution. The program has the capability to feed its output (results) of one segment into the next segment in the form of initial conditions where the states and their probabilities are exactly specified. Recall that no independence assumption was made in evaluating the pollution phase of the program but the assumption was made

concerning the dissolved oxygen probabilities of states, means and variances. The assumption of independence (more extensively discussed in the next chapter) does not seem to significantly alter the results.

The salient features of the computer program are as follows:

1. It allows discrete changes in any or all stream parameters. All that is necessary is a computer data card with the changes and the "old" data that is not to be changed.

2. The program analyzes major downstream polluting sources. In this case the states and their probabilities at the polluting source are no longer the same as they were prior to being polluted. To continue the stream analysis, the program needs to know the BOD concentration at the major polluting source.

3. The program can print some intermediate results at any points within a homogeneous stream segment to see what the BOD and DO distributions look like. This printing does not alter the results of any part of the stream analysis in any manner. The intermediate information is purely supplementary. The command to monitor the intermediate or final results of a segment is a code: one (1) has the program print intermediate results; and zero (0) prints final results of stream segments.

The flow of the program is in three phases: a pollution phase, a dissolved oxygen phase, and a phase assigning the output probabilities of one segment to the initial probabilities of the suceeding segment (when applicable).

The pollution phase first computes the probability distribution as high of a concentration as the initial pollution and prints the non-zero probabilities with their corresponding concentrations. Secondly, for a given $a$ confidence level, the program sums the individual probabilities beginning with those corresponding to the upper concentrations of pollution. It prints the concentration and the cumulative probability for the nearest cumulative probability greater than $a$, and the nearest cumulative probability less than $a$. This gives an upper confidence limit on pollution. Thirdly, it evaluates and prints the mean and variance of the amount of pollution. This concludes the first phase.

The second phase is working with dissolved oxygen for the same time of travel. First it calculates the probability distribution as high as the maximum DO concentration and prints the non-zero probabilities with their corresponding concentrations. Secondly it sums the probabilities starting with the smallest DO concentration until it reaches the $\alpha$ confidence limit. It then prints the cumulative probability and the nearest cumulative probability greater than $\alpha$ and the nearest cumulative less than $\alpha$. This gives a lower confidence limit on dissolved oxygen. Thirdly, it evaluates and prints the mean and variance of the amount of dissolved oxygen, in this way concluding the second phase of the program.

The third phase (when applicable) equates the final computed probabilities of BOD and DO to the initial probabilities of the next stream segment. Of course if the results are only intermediate information within a homogeneous segment, then this third phase is by-passed so no harm will be done to the succeeding stream segments.

The program also has other features. For example in the equation for the probability of the $M^{th}$ state of pollution (3.2.11) there is an infinite sum of probabilities multiplied by some other weighting factors. Instead of having the program compute each of the infinite sums over a very large range of values (maybe 500 of them), the program was adjusted to search for the highest probability used. This precludes any unnecessary work. Another feature is that it checks the parameters to be sure that the computer will not have to divide by zero. If $K_2 = (K_1 + K_3)$ in the expression for $N^{th}$ state of DO (3.2.13), the computer would not be able to evaluate it. Therefore, the program rejects the data card with the unusual parameter values and tells the programmer what the problem is.

The program that is listed in Appendix I is written in Fortran IV language and requires three subroutines, several control cards, and a card to permit underflows. The three subroutines, also listed in Appendix I, are necessary inclusions to solve the probabilities for both BOD and DO. The control cards were omitted from the example program since each computer has a different requirement in this respect. These control cards are usually necessary at the beginning and end of the program and prior to the subroutines. In addition to the above cards, one must have a card which permits underflows in the computer (EXTERNAL NOLMUN card is used in the programs in Appendix I and II). A similar card is necessary since some of the probabilities that the machine computes are obviously quite small.

The values needed for the input data cards are CODE, TIME, $K_1$, $K_2$, $K_3$, $L_a$, $D_B$, $\Delta$, $\alpha$, L, $\frac{C_{N'}}{\Delta}$, and D. All twelve values have to occur

consecutively on the same data card for each set of data. Each parameter value is allotted a certain number of spaces (see Table 1) in which to be entered, and there should be no spacing between the parameter allotments. The decimal point in the Floating Point numbers must be placed in one of the six spaces. The Fixed Point numbers have no decimals, but the numbers have to be placed as far to the right as possible in their own allotted spaces to prevent the computer from misinterpreting the desired values.

It would be well to explain all the terms used in the computer program to avoid confusion. These twelve terms appear on the data cards:

1. CODE is used to determine if the machine is supposed to print the results at the end of a stream segment (CODE = zero) or if it is to print intermediate results within a stream segment (CODE = one).

2. TIME is the time of travel (in units of day).

3. $K_1$ is the rate constant associated with the action of bacteria (in units of per day).

4. $K_2$ is the rate constant associated with reaeration (in units of per day).

5. $K_3$ is the rate constant associated with sedimentation and adsorption of the pollution (in units of per day).

6. $L_a$ is the rate constant associated with the small sources of pollution along the sides (in units of ppm per day).

7. $D_B$ is the rate constant associated with Benthal Demand (in units of ppm per day).

8. $\Delta$ is the parameter for the stream which is obtained from the variance (in units of ppm per state).

9. $a$ is the confidence level which is set by the experimenter (no units).

10. $L_A$ is the mean initial pollution concentration (in units of ppm) and is used in the program in integer form $\frac{L_A}{\Delta}$ which has conveniently been called L.

Table 1. Arrangement of Data Cards

| PARAMETER | PROGRAM'S NOTATION | NUMBER OF SPACES | |
|---|---|---|---|
| CODE | KODE | 1 | Fixed Point |
| TIME | TIME | 6 | Floating Point |
| $K_1$ | ZK1 | 6 | Floating Point |
| $K_2$ | ZK2 | 6 | Floating Point |
| $K_3$ | ZK3 | 6 | Floating Point |
| $L_a$ | ZLA | 6 | Floating Point |
| $D_B$ | ZDB | 6 | Floating Point |
| $\Delta$ | *DELTA* | 6 | Floating Point |
| $\alpha$ | ALPHA | 6 | Floating Point |
| L | I | 4 | Fixed Point |
| $\frac{C_N'}{\Delta}$ | NP | 4 | Fixed Point |
| D | J | 4 | Fixed Point |

11. $C_{N'}$ is the maximum oxygen solubility for given conditions of temperature and barometric pressure (in units of ppm) and is used in the program in integer form $\frac{C_{N'}}{\Delta}$.

12. $C_o$ is the mean initial dissolved oxygen concentration (in units of ppm) and is used in the program in integer form $\frac{C_o}{\Delta}$ which has conveniently been called D.

Realizing that probabilities must sum to unity, one might notice that sometimes the probabilities for pollution and dissolved oxygen sum to something less than unity. This shortcoming is due to the fact that the range of pollution and dissolved oxygen is from zero to the input value. The mathematics, however, allows for an extended range which is not physically meangful. Nevertheless, this decrease of total probability is very small and will be neglected.

## STUDY OF INDEPENDENCE

### Correlation Coefficient Study

The assumption of independence was made in the cases of the probability generating function for dissolved oxygen (3.2.12), the theoretical mean amount of DO (3.3.6) and the theoretical variance in amount of DO (3.3.7). In the computer program, the sample mean and sample variance of DO was computed and printed since it was noticed that the theoretical means and variances of DO were not exactly correct. Therefore, the only equation that uses the independence assumption in the program is the DO generating function which gives rise to the dissolved oxygen probabilities.

From the chemistry of the problem and from preliminary computer simulations, it was known that the BOD and DO were not strictly independent. The next step was to determine if the two factors were sufficiently unrelated to consider them as being statistically independent. The method of correlation coefficients was chosen to make this determination.

The model used for the correlation coefficient test was the reduced model of Streeter and Phelps (5). This model is similar to that of Thayer and Krutchkoff (6) and Dobbins (2) with the restriction that $L_a = D_B = K_3 = 0$. The simplified model was merely chosen to reduce mathematics in deriving the equations.

The expression for the correlation coefficient between BOD and DO is

$$\text{Correl. coeff.} = \frac{\text{Covariance}}{\sqrt{(\text{BOD variance})\,(\text{DO variance})}} \qquad (4.1.1)$$

When solving the above expression, the following joint generating function is used:

$$P(S,R;t) = \left[\left[1 + \left(\frac{1-R}{R}\right) e^{-K_2 t}\right]\right]^{\frac{C_{N'}}{\Delta} - D_{.R}} \frac{C_{N'}}{\Delta}$$

$$\cdot \left[1 + \frac{K_1}{K_2 - K_1} \left(\frac{1-R}{R}\right) \left(e^{-K_1 t} - e^{-K_2 t}\right)\right]$$

$$- (1-S)\, e^{-K_1 t} \Big] \qquad (4.1.2)$$

which is a simplified form of the generating function in Thayer and Krutchkoff. The means and variances of amounts of BOD and DO from the section, Stochastic Model, are used with the restriction mentioned above. The covariance term is derived from the mixed partial derivative with respect to S and R of the joint generating function and is evaluated at S = R = 1. Then for covariance of the amount, one obtains

$$\left.\frac{\partial P\,(S,R;t)}{\partial S\,\partial R}\right|_{\substack{S=1\\R=1}} = \Delta^2 \Big[ L\,(L-1) \cdot \frac{K_1}{K_2 - K_1}\ (e^{-K_2 t} - e^{-K_1 t})$$

$$\cdot e^{-K_1 t} + L \cdot \frac{C_{N'}}{\Delta} \cdot e^{-K_1 t} - L \left(\frac{C_{N'}}{\Delta} - D\right)$$

$$\cdot e^{-K_1 t} \cdot e^{-K_2 t} \Big] \qquad (4.1.3)$$

The correlation analysis was made on three variables: $K_1$, the bacteria rate constant; $K_2$, the reaeration rate constant; and t, the time of travel. The ranges for $K_1$ and $K_2$ were selected from the realistic values that Kothandaraman (4) described. The values of $K_1$ were chosen to range from 0.1 to 0.5 in increments of 0.1. The $K_2$ was allowed to range from 0.2 to 3.0 in increments of 0.2. The time factor ranged from one day to 5 days in increments of integer days. Other parameter values were arbitrarily selected and were as follows:

$$\frac{L_A}{\Delta} = L = 124 \qquad \frac{C_{N'}}{\Delta} = 104$$

$$\frac{C_o}{\Delta} = D = 50 \qquad \text{delta} = 0.1$$

The test was constructed in the above manner and the program was written (Appendix III) to handle the arithmetic.

The results of the correlation coefficient study were difficult to interpret. The computer results appear in Tables 2, 3, 4, 5, and 6. Under certain conditions the correlation coefficient value was sufficiently close to zero to conclude the variables are nearly independent. This general conclusion can be made from Table 3 for large values of $K_1$ and $K_2$ and from the majority of the combinations in the succeeding Tables (4, 5, 6). This partial independence can be attributed to the fact that about the third day the phenomenon of the "oxygen curve sag" occurs. This drop in dissolved oxygen concentration occurs when the available oxygen readily reacts with a large quantity of pollution molecules. However, the correlation study is not very conclusive in illustrating independence as well as it illustrates the known fact of dependence of BOD and DO.

## Results of Independence Assumption

The next step in the study of independence was to analyze the original program of a homogeneous stream stretch with fixed point initial conditions and compare the results to the generalized program under the same conditions in Appendix I where the assumption of independence is made on the dissolved oxygen probabilities. This determination was made and the results appeared quite favorable.

The two programs were compared by making BOD and DO analysis for 5 integer days. The original program did not use the independence assumption nor did it consider that the parameters might appreciably change within certain parts of the stream within 5 days. The generalized program in Appendix I did make the independence assumption only for the dissolved oxygen probabilities. (Recall that the sample means and variances for DO were used instead of the theoretical values which assumed independence). Further, the generalized program analytically dissected the 5 day stream into 5 stream segments, each containing one day of travel; no parameter changes were made nor was any pollution added downstream.

Table 2. Correlation Coefficients of $K_1$ Versus $K_2$ at t = 1 Day

| $K_1$ \ $K_2$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | .70 | .59 | .52 | .49 | .46 | .45 | .43 | .43 | .42 | .42 | .41 | .41 | .41 | .41 | .41 |
| 0.2 | ___* | .69 | .61 | .56 | .53 | .51 | .50 | .48 | .47 | .46 | .46 | .45 | .44 | .44 | .43 |
| 0.3 | .80 | .70 | .63 | .59 | .56 | .53 | .51 | .50 | .48 | .47 | .46 | .45 | .44 | .44 | .43 |
| 0.4 | .81 | ___ | .64 | .60 | .56 | .53 | .51 | .50 | .48 | .47 | .46 | .44 | .43 | .43 | .42 |
| 0.5 | .81 | .71 | .64 | .59 | .56 | .53 | .51 | .49 | .47 | .46 | .44 | .43 | .42 | .41 | .40 |

*In this and succeeding tables where $K_1 = K_2$, the correlation coefficient is undefined. See equations (2.2.6; 2.2.9) for the variances when $L_a = D_B = K_3 = 0$.

Table 3. Correlation Coefficients of $K_1$ Versus $K_2$ at t = 2 Days

| $K_1$ \ $K_2$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | .67 | .56 | .51 | .48 | .46 | .45 | .44 | .43 | .42 | .41 | .40 | .39 | .38 | .37 | .35 |
| 0.2 | — | .60 | .53 | .50 | .47 | .44 | .43 | .41 | .39 | .38 | .36 | .35 | .34 | .33 | .32 |
| 0.3 | .70 | .58 | .52 | .47 | .44 | .41 | .39 | .37 | .36 | .34 | .32 | .31 | .30 | .29 | .28 |
| 0.4 | .68 | — | .49 | .41 | .41 | .38 | .36 | .34 | .32 | .30 | .29 | .27 | .26 | .25 | .24 |
| 0.5 | .65 | .53 | .45 | .41 | .37 | .34 | .37 | .30 | .28 | .27 | .25 | .24 | .23 | .22 | .21 |

Table 4. Correlation Coefficients of $K_1$ Versus $K_2$ at t = 3 Days

| $K_1$ \ $K_2$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | .63 | .53 | .48 | .45 | .43 | .41 | .39 | .37 | .35 | .34 | .32 | .31 | .29 | .28 | .27 |
| 0.2 | — | .52 | .46 | .41 | .38 | .36 | .33 | .31 | .29 | .28 | .36 | .25 | .24 | .23 | .22 |
| 0.3 | .60 | .47 | .41 | .36 | .33 | .30 | .28 | .26 | .24 | .23 | .22 | .24 | .20 | .19 | .18 |
| 0.4 | .54 | — | .35 | .31 | .27 | .25 | .23 | .21 | .20 | .19 | .17 | .17 | .16 | .15 | .14 |
| 0.5 | .49 | .36 | .30 | .23 | .21 | .19 | .19 | .17 | .16 | .15 | .14 | .13 | .13 | .12 | .12 |

Table 5. Correlation Coefficients of $K_1$ Versus $K_2$ at t = 4 Days

| $K_1$ \ $K_2$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | .60 | .50 | .44 | .41 | .38 | .35 | .33 | .31 | .29 | .27 | .26 | .25 | .24 | .23 | .22 |
| 0.2 | —— | .44 | .38 | .34 | .30 | .27 | .25 | .23 | .22 | .21 | .20 | .19 | .18 | .17 | .16 |
| 0.3 | .49 | .37 | .31 | .27 | .24 | .21 | .19 | .18 | .17 | .16 | .15 | .14 | .13 | .13 | .12 |
| 0.4 | .41 | —— | .24 | .21 | .18 | .16 | .15 | .13 | .12 | .12 | .11 | .10 | .10 | .09 | .09 |
| 0.5 | .35 | .24 | .19 | .16 | .14 | .12 | .11 | .10 | .09 | .09 | .08 | .08 | .07 | .07 | .07 |

Table 6. Correlation Coefficients of $K_1$ Versus $K_2$ at t = 5 Days

| $K_1$ \ $K_2$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 | 2.2 | 2.4 | 2.6 | 2.8 | 3.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | .56 | .46 | .40 | .36 | .32 | .30 | .27 | .25 | .24 | .23 | .21 | .20 | .20 | .19 | .18 |
| 0.2 | — | .37 | .31 | .27 | .24 | .21 | .19 | .18 | .17 | .16 | .15 | .14 | .14 | .13 | .13 |
| 0.3 | .40 | .29 | .23 | .19 | .17 | .15 | .14 | .12 | .12 | .11 | .10 | .10 | .09 | .09 | .09 |
| 0.4 | .31 | — | .17 | .14 | .12 | .10 | .09 | .09 | .08 | .07 | .07 | .07 | .06 | .06 | .06 |
| 0.5 | .24 | .16 | .12 | .10 | .08 | .07 | .06 | .06 | .05 | .05 | .05 | .04 | .04 | .04 | .04 |

The values used in the comparison were as follows:

$K_1$ = 0.35 per day

$K_2$ = 0.75 per day

$K_3$ = 0.20 per day

$L_a$ = 0.20 ppm/day

$D_B$ = 0.10 ppm/day

delta = 0.10 ppm/state

alpha = 0.10

$L_A$ = 10.0 ppm

$C_{N'}$ = 12.1 ppm

$C_o$ = 8.0 ppm

The original main program (with modifications) is included in Appendix II for convenience while the three necessary subroutines are included in Appendix I. The main program was modified to handle the more generalized WEIGHTED BINOMIAL SUBROUTINE.

The comparison of the two programs indicated that the pollution probabilities were exactly the same for the corresponding states and that the dissolved oxygen probabilities were very close. Further, the comparison showed that the respective means and variances were the same in most of the cases.

For the first day, both the BOD probabilities and the DO probabilities were respectively the same when compared. In addition, the respective means and variances were alike. This can be attributed to the fact that both programs considered the same initial conditions for the first day, that is a point source of pollution. Hence, both programs solved the same equations. For the second and succeeding days, it is understandable and expected that the BOD probabilities, means and variances were the same since no independence was assumed and since one program was a generalization of the other. However, the DO probabilities and variances (not including the means) after the 1st day of comparisons were not exactly the same because of the lack of independence. In all cases, it was noted that in comparison of BOD or DO, the means were exactly the same in comparison regardless of time frame.

To graphically illustrate some of the results, the curves were drawn comparing both programs. Figure 2 illustrates the fact that the BOD probabilities are indeed the same at t = 1 as an example. For increasing time of travel, the BOD curves actually coincided. The Figures 3, 4, 5, 6, and 7 depict the dissolved oxygen probability curves. The points nearly coincided with the curves and the means are exactly the same. The curves illustrate that

the assumption of independence did not harm the analysis in any appreciable way. Thus the DO distributions given are effective solutions to the problem.

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - BOD
TIME = 1.0

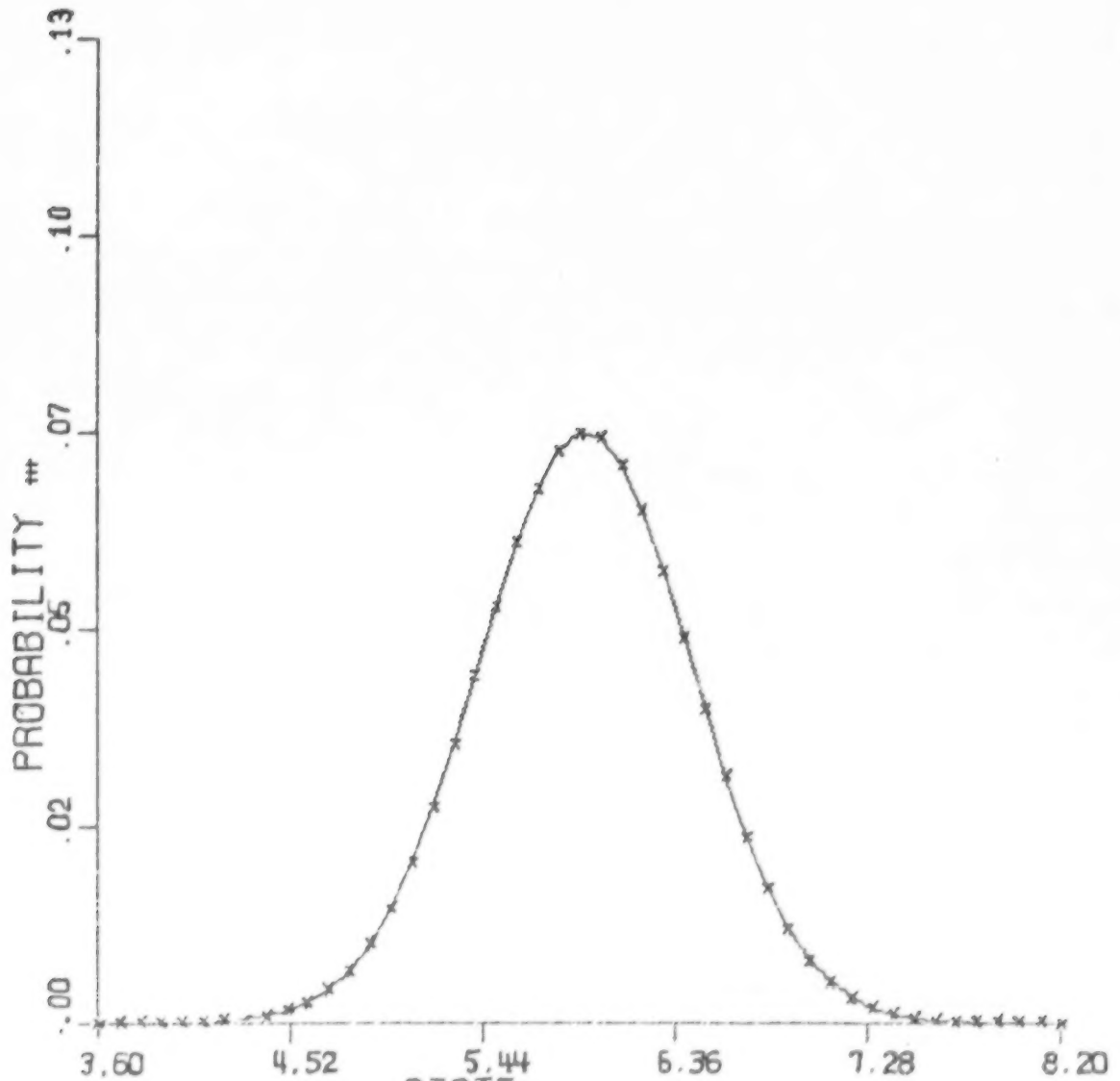


FIGURE 1

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - D0
TIME = 1.0

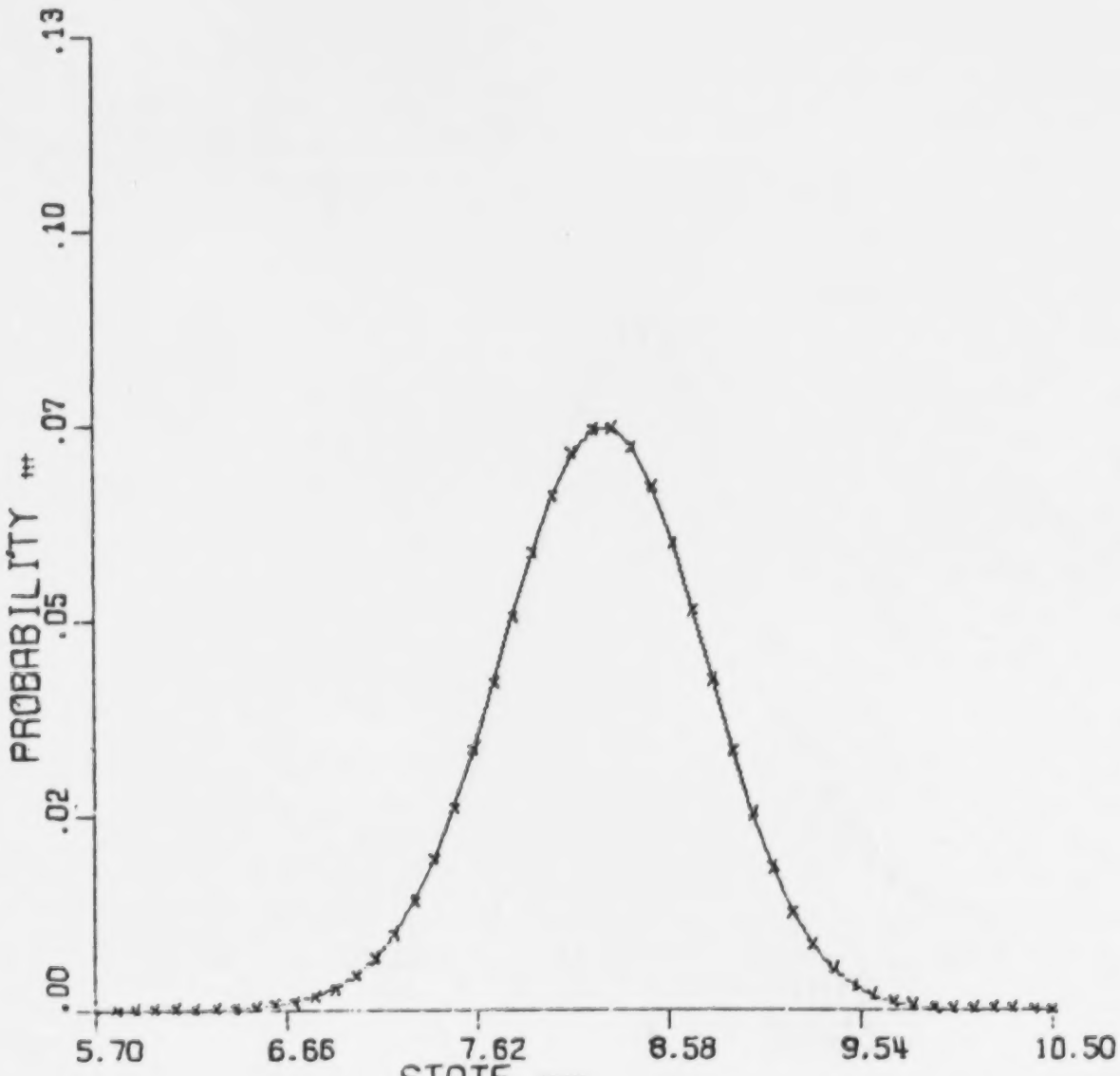


FIGURE 2

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - DO
TIME = 2.0

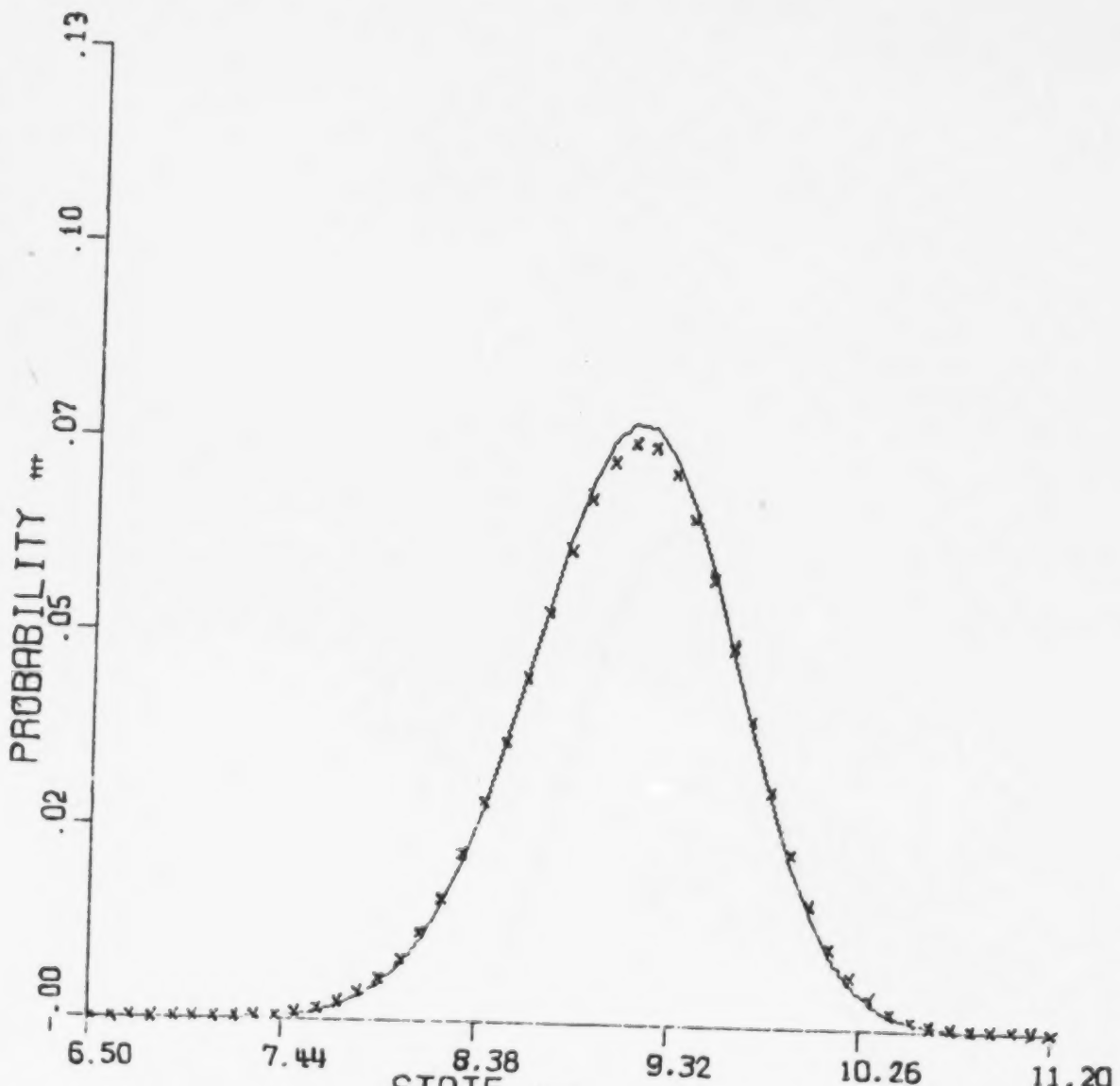


FIGURE 3

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - D0
TIME = 3.0

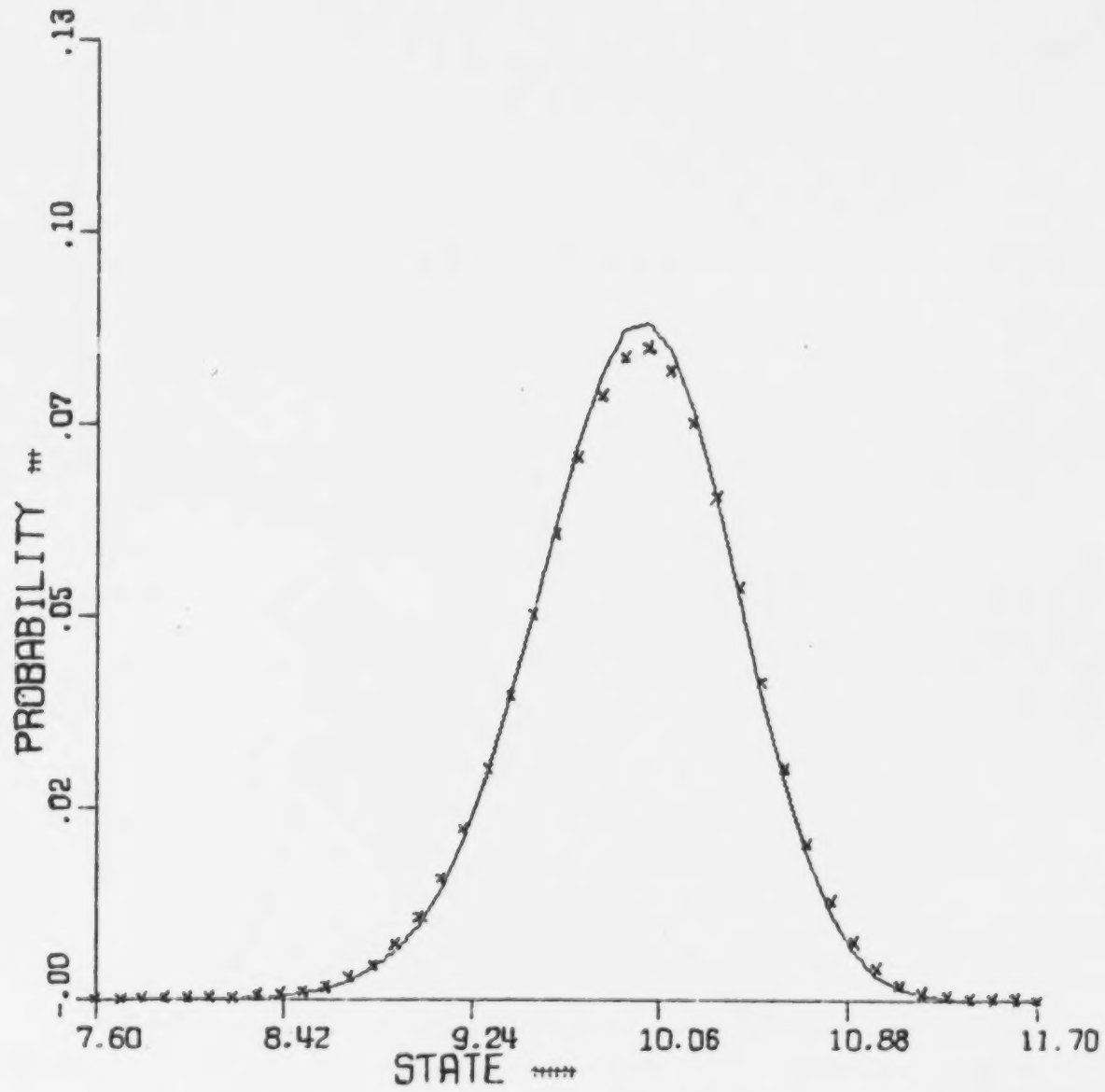


FIGURE 4

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - D0
TIME = 4.0

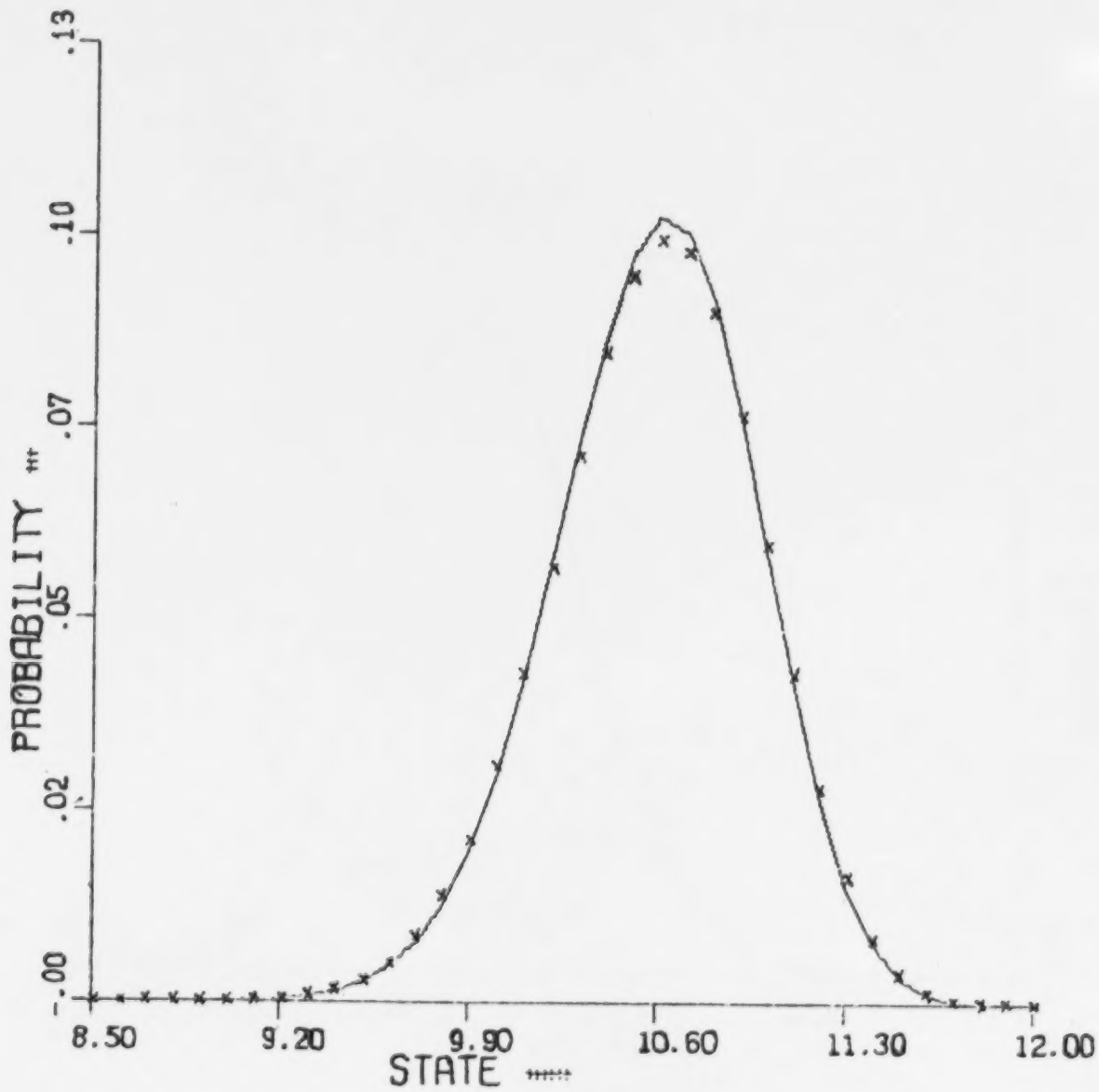


FIGURE 5

COMPARING INITIAL PT SOURCE(LINE)
TO A DIST SOURCE (X) - D0
TIME = 5.0

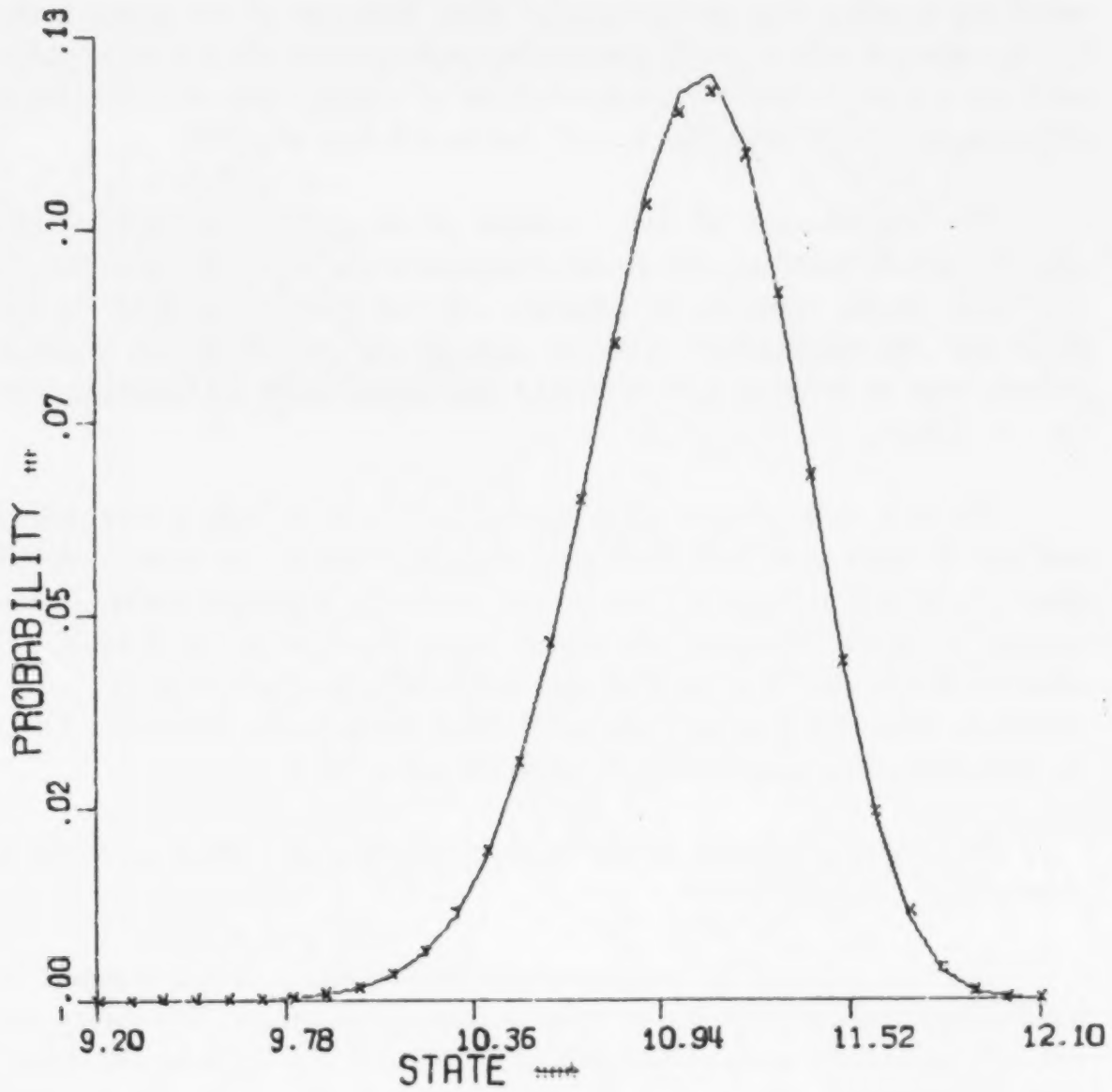


FIGURE 6

# EXAMPLE

This chapter includes a hypothetical example whose purpose is to illustrate the capability of the computer program (in Appendix I). The example does not use actual data but the parameter values were chosen in a realistic range as depicted by Kothandaraman (4).

The stream stretch of interest begins at some point downstream where the BOD and DO are measured at a point source of pollution. The water flows relatively rapid throughout a known distance where the average time of travel is .6 days. This rapid part of the stream will be denoted as the first stream segment. At the end of t = .6 days the river broadens in depth and width for another time of travel of 2.5 days. This part of the stream is the second segment. After t = 2.5 days another point source of pollution is added with known BOD and DO concentrations of effluent and the succeeding stream segment (the third one) is analyzed for 2.0 days of travel.

The requirement of the example is to analyze the probability distributions of pollution and dissolved oxygen at the end of the first, second and third stream segments. In addition, take intermediate readings on the BOD and DO distributions half-way through the second stream segment (whose time of travel is 1.25 days) and half-way through the third segment (at t = 1.0 day).

The data to be printed on 6 data cards is listed in Table 8. The second and fourth data cards indicate that intermediate results are to be obtained since the CODE is equal to one in both cases (as explained earlier in the section, Computer Program Discussion). Note that the last card has t = 0 which indicates that it is the final data card. After the machine has solved all equations using the previous data cards, then the program terminates. Since no actual data was used, $\Delta$ was arbitrarily chosen to be 0.1.

The computer results of the hypothetical example follow in Tables 9 through 18.

The true value of the program lies in its versatility. It was designed to handle large sources of pollution at some point downstream. This ability was not reflected in the work of Thayer and Krutchkoff. Furthermore the stream parameters did change at certain locations where the concept of homogeneous stream segments was introduced. In addition, the concept of independence of BOD and DO was not detrimental to the results (as was

indicated in the section, Study of Independence). The independence of the problem was critical in placing the results of one stream segment into the next segment in the form of initial conditions.

Table 7. Hypothetical Stream Data

| Stream Segment | CODE | Time of Travel (Days) | $K_1$ (per day) | $K_2$ (per day) | $K_3$ (per day) | $L_a$ (ppm/day) | $D_B$ (ppm/day) | $\Delta$ (ppm/state) | $\alpha$ | L (ppm) | $\frac{C_{N'}}{\Delta}$ (ppm) | D (ppm) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FIRST | 0 | 0.6 | 0.12 | 3.0 | -0.01 | 0.2 | 0.5 | 0.1 | 0.1 | 6.8 | 12.1 | 8.0 |
| SECOND | 1 | 1.25 | 0.12 | 0.6 | 0.2 | 0.2 | 0.5 | 0.1 | 0.1 | 6.8 | 12.1 | 8.0 |
| SECOND | 0 | 2.5 | 0.12 | 0.6 | 0.2 | 0.2 | 0.5 | 0.1 | 0.1 | 6.8 | 12.1 | 8.0 |
| THIRD | 1 | 1.0 | 0.12 | 0.6 | 0.2 | 0.2 | 0.5 | 0.1 | 0.1 | 10.5 | 12.1 | 8.0 |
| THIRD | 0 | 2.0 | 0.12 | 0.6 | 0.2 | 0.2 | 0.5 | 0.1 | 0.1 | 10.5 | 12.1 | 8.1 |

Table 8. BOD Results of Segment 1 where t = .6 Days

RESULTS (POLLUT. AND DISS. OXYGEN DENSITIES)
AT THE END OF STREAM SEGMENT FOR

STREAM SEGMENT = 1
TIME INTERVAL = 0.600 DAYS

ZK1 (BACTERIA COEF) = 0.120 PER DAY
ZK2 (REAERATION COEF) = 3.000 PER DAY
ZK3 (SEDIM. AND ADSORP. COEF) = -0.010 PER DAY
ZLA (LAND RUNOFF COEF) = 0.200 PPM PER DAY
ZDB (BENTHAL DEMAND) = 0.500 PPM PER DAY
ALPHA LEVEL = 0.100
I (INITIAL POLLUTION) = 6.800 PPM PER DAY
NP (MAX DO) = 12.100 PPM PER DAY
J (INITIAL DO) = 8.000 PPM PER DAY

PROBABILITY MASS DENSITIES FOR POLLUTION
PROBABILITY OF HAVING 5.20 PPM = 0.00000
PROBABILITY OF HAVING 5.30 PPM = 0.00001
PROBABILITY OF HAVING 5.40 PPM = 0.00003
PROBABILITY OF HAVING 5.50 PPM = 0.00010
PROBABILITY OF HAVING 5.60 PPM = 0.00037
PROBABILITY OF HAVING 5.70 PPM = 0.00116
PROBABILITY OF HAVING 5.80 PPM = 0.00337
PROBABILITY OF HAVING 5.90 PPM = 0.00877
PROBABILITY OF HAVING 6.00 PPM = 0.02041
PROBABILITY OF HAVING 6.10 PPM = 0.04207
PROBABILITY OF HAVING 6.20 PPM = 0.07600
PROBABILITY OF HAVING 6.30 PPM = 0.11886
PROBABILITY OF HAVING 6.40 PPM = 0.15847
PROBABILITY OF HAVING 6.50 PPM = 0.17689
PROBABILITY OF HAVING 6.60 PPM = 0.16183
PROBABILITY OF HAVING 6.70 PPM = 0.11855
PROBABILITY OF HAVING 6.80 PPM = 0.06808
PROBABILITY OF HAVING 6.90 PPM = 0.03039

TOTAL PROBABILITIES = 0.98537

MEAN = 6.38 VARIANCE = 0.6519

PROBABILITY OF HAVING MORE THAN 6.80 PPM = 0.0450
PROBABILITY OF HAVING MORE THAN 6.70 PPM = 0.1131

Table 9. DO Results of Segment 1 Where t = .6 Days

STREAM SEGMENT = 1
TIME INTERVAL = 0.600 DAYS

PROBABILITY MASS DENSITIES FOR OXYGEN

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING | 12.10 | PPM = | 0.00002 |
| PROBABILITY OF HAVING | 12.00 | PPM = | 0.00019 |
| PROBABILITY OF HAVING | 11.90 | PPM = | 0.00110 |
| PROBABILITY OF HAVING | 11.80 | PPM = | 0.00423 |
| PROBABILITY OF HAVING | 11.70 | PPM = | 0.01203 |
| PROBABILITY OF HAVING | 11.60 | PPM = | 0.02699 |
| PROBABILITY OF HAVING | 11.50 | PPM = | 0.04985 |
| PROBABILITY OF HAVING | 11.40 | PPM = | 0.07794 |
| PROBABILITY OF HAVING | 11.30 | PPM = | 0.10532 |
| PROBABILITY OF HAVING | 11.20 | PPM = | 0.12494 |
| PROBABILITY OF HAVING | 11.10 | PPM = | 0.13173 |
| PROBABILITY OF HAVING | 11.00 | PPM = | 0.12469 |
| PROBABILITY OF HAVING | 10.90 | PPM = | 0.10684 |
| PROBABILITY OF HAVING | 10.80 | PPM = | 0.08344 |
| PROBABILITY OF HAVING | 10.70 | PPM = | 0.05976 |
| PROBABILITY OF HAVING | 10.60 | PPM = | 0.03944 |
| PROBABILITY OF HAVING | 10.50 | PPM = | 0.02410 |
| PROBABILITY OF HAVING | 10.40 | PPM = | 0.01368 |
| PROBABILITY OF HAVING | 10.30 | PPM = | 0.00724 |
| PROBABILITY OF HAVING | 10.20 | PPM = | 0.00359 |
| PROBABILITY OF HAVING | 10.10 | PPM = | 0.00167 |
| PROBABILITY OF HAVING | 10.00 | PPM = | 0.00073 |
| PROBABILITY OF HAVING | 9.90 | PPM = | 0.00030 |
| PROBABILITY OF HAVING | 9.80 | PPM = | 0.00012 |
| PROBABILITY OF HAVING | 9.70 | PPM = | 0.00004 |
| PROBABILITY OF HAVING | 9.60 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 9.50 | PPM = | 0.00000 |
| PROBABILITY OF HAVING | 9.40 | PPM = | 0.00000 |

TOTAL PROBABILITIES = 1.00000

MEAN = 11.06 VARIANCE = 0.0918

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING LESS THAN | 10.80 | PPM = | 0.1507 |
| PROBABILITY OF HAVING LESS THAN | 10.70 | PPM = | 0.0909 |

Table 10. BOD Results of Segment 2 Where t = 1.25 Days

INTERMED. RESULTS (POLLUT. AND DISS.OXYGEN DENSITIES) FOR

STREAM SEGMENT = 2
TIME INTERVAL = 1.250 DAYS

ZK1 (BACTERIA COEF) = 0.120 PER DAY
ZK2 (REAERATION COEF) = 0.600 PER DAY
ZK3 (SEDIM. AND ADSORP. COEF) = 0.200 PER DAY
ZLA (LAND RUNOFF COEF) = 0.200 PPM PER DAY
ZDB (BENTHAL DEMAND) = 0.100 PPM PER DAY
ALPHA LEVEL = 0.100
I (INITIAL POLLUTION) = 6.800 PPM PER DAY
NP (MAX DO) = 12.100 PPM PER DAY
J (INITIAL DO) = 8.000 PPM PER DAY

PROBABILITY MASS DENSITIES FOR POLLUTION

| | | |
|---|---|---|
| PROBABILITY OF HAVING | 2.50 PPM | = 0.00000 |
| PROBABILITY OF HAVING | 2.60 PPM | = 0.00000 |
| PROBABILITY OF HAVING | 2.70 PPM | = 0.00001 |
| PROBABILITY OF HAVING | 2.80 PPM | = 0.00003 |
| PROBABILITY OF HAVING | 2.90 PPM | = 0.00007 |
| PROBABILITY OF HAVING | 3.00 PPM | = 0.00016 |
| PROBABILITY OF HAVING | 3.10 PPM | = 0.00035 |
| PROBABILITY OF HAVING | 3.20 PPM | = 0.00073 |
| PROBABILITY OF HAVING | 3.30 PPM | = 0.00145 |
| PROBABILITY OF HAVING | 3.40 PPM | = 0.00275 |
| PROBABILITY OF HAVING | 3.50 PPM | = 0.00491 |
| PROBABILITY OF HAVING | 3.60 PPM | = 0.00834 |
| PROBABILITY OF HAVING | 3.70 PPM | = 0.01342 |
| PROBABILITY OF HAVING | 3.80 PPM | = 0.02048 |
| PROBABILITY OF HAVING | 3.90 PPM | = 0.02966 |
| PROBABILITY OF HAVING | 4.00 PPM | = 0.04076 |
| PROBABILITY OF HAVING | 4.10 PPM | = 0.05313 |
| PROBABILITY OF HAVING | 4.20 PPM | = 0.06571 |
| PROBABILITY OF HAVING | 4.30 PPM | = 0.07708 |
| PROBABILITY OF HAVING | 4.40 PPM | = 0.08577 |
| PROBABILITY OF HAVING | 4.50 PPM | = 0.09050 |
| PROBABILITY OF HAVING | 4.60 PPM | = 0.09052 |
| PROBABILITY OF HAVING | 4.70 PPM | = 0.08582 |
| PROBABILITY OF HAVING | 4.80 PPM | = 0.07708 |
| PROBABILITY OF HAVING | 4.90 PPM | = 0.06556 |
| PROBABILITY OF HAVING | 5.00 PPM | = 0.05279 |
| PROBABILITY OF HAVING | 5.10 PPM | = 0.04021 |
| PROBABILITY OF HAVING | 5.20 PPM | = 0.02897 |
| PROBABILITY OF HAVING | 5.30 PPM | = 0.01972 |
| PROBABILITY OF HAVING | 5.40 PPM | = 0.01268 |
| PROBABILITY OF HAVING | 5.50 PPM | = 0.00770 |
| PROBABILITY OF HAVING | 5.60 PPM | = 0.00441 |
| PROBABILITY OF HAVING | 5.70 PPM | = 0.00238 |
| PROBABILITY OF HAVING | 5.80 PPM | = 0.00121 |
| PROBABILITY OF HAVING | 5.90 PPM | = 0.00058 |
| PROBABILITY OF HAVING | 6.00 PPM | = 0.00026 |
| PROBABILITY OF HAVING | 6.10 PPM | = 0.00011 |
| PROBABILITY OF HAVING | 6.20 PPM | = 0.00004 |
| PROBABILITY OF HAVING | 6.30 PPM | = 0.00002 |
| PROBABILITY OF HAVING | 6.40 PPM | = 0.00001 |
| PROBABILITY OF HAVING | 6.50 PPM | = 0.00000 |

TOTAL PROBABILITIES = 0.98537

MEAN = 4.48 VARIANCE = 0.4806

| | | |
|---|---|---|
| PROBABILITY OF HAVING MORE THAN | 5.10 PPM | = 0.0927 |
| PROBABILITY OF HAVING MORE THAN | 5.00 PPM | = 0.1329 |

Table 11. DO Results of Segment 2 Where t = 1.25 Days

STREAM SEGMENT = 2
TIME INTERVAL = 1.250 DAYS

PROBABILITY MASS DENSITIES FOR OXYGEN

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING | 12.10 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 12.00 | PPM = | 0.00009 |
| PROBABILITY OF HAVING | 11.90 | PPM = | 0.00056 |
| PROBABILITY OF HAVING | 11.80 | PPM = | 0.00225 |
| PROBABILITY OF HAVING | 11.70 | PPM = | 0.00672 |
| PROBABILITY OF HAVING | 11.60 | PPM = | 0.01598 |
| PROBABILITY OF HAVING | 11.50 | PPM = | 0.03149 |
| PROBABILITY OF HAVING | 11.40 | PPM = | 0.05287 |
| PROBABILITY OF HAVING | 11.30 | PPM = | 0.07723 |
| PROBABILITY OF HAVING | 11.20 | PPM = | 0.09972 |
| PROBABILITY OF HAVING | 11.10 | PPM = | 0.11523 |
| PROBABILITY OF HAVING | 11.00 | PPM = | 0.12036 |
| PROBABILITY OF HAVING | 10.90 | PPM = | 0.11458 |
| PROBABILITY OF HAVING | 10.80 | PPM = | 0.10012 |
| PROBABILITY OF HAVING | 10.70 | PPM = | 0.08077 |
| PROBABILITY OF HAVING | 10.60 | PPM = | 0.06046 |
| PROBABILITY OF HAVING | 10.50 | PPM = | 0.04219 |
| PROBABILITY OF HAVING | 10.40 | PPM = | 0.02755 |
| PROBABILITY OF HAVING | 10.30 | PPM = | 0.01689 |
| PROBABILITY OF HAVING | 10.20 | PPM = | 0.00975 |
| PROBABILITY OF HAVING | 10.10 | PPM = | 0.00532 |
| PROBABILITY OF HAVING | 10.00 | PPM = | 0.00275 |
| PROBABILITY OF HAVING | 9.90 | PPM = | 0.00135 |
| PROBABILITY OF HAVING | 9.80 | PPM = | 0.00063 |
| PROBABILITY OF HAVING | 9.70 | PPM = | 0.00028 |
| PROBABILITY OF HAVING | 9.60 | PPM = | 0.00012 |
| PROBABILITY OF HAVING | 9.50 | PPM = | 0.00005 |
| PROBABILITY OF HAVING | 9.40 | PPM = | 0.00002 |
| PROBABILITY OF HAVING | 9.30 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 9.20 | PPM = | 0.00000 |

TOTAL PROBABILITIES = 0.98535

MEAN = 10.80 VARIANCE = 1.8392

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING LESS THAN | 10.80 | PPM = | 0.1069 |
| PROBABILITY OF HAVING LESS THAN | 10.50 | PPM = | 0.0647 |

Table 12. BOD Results of Segment 2 Where t = 2.50 Days

RESULTS (POLLUT. AND DISS. OXYGEN DENSITIES)
AT THE END OF STREAM SEGMENT FOR

STREAM SEGMENT = 2
TIME INTERVAL = 2.500 DAYS

ZK1 (BACTERIA COEF) = 0.120 PER DAY
ZK2 (REAERATION COEF) = 0.600 PER DAY
ZK3 (SEDIM. AND ADSORP. COEF) = 0.200 PER DAY
ZLA (LAND RUNOFF COEF) = 0.200 PPM PER DAY
ZDB (BENTHAL DEMAND) = 0.100 PPM PER DAY
ALPHA LEVEL = 0.100
I (INITIAL POLLUTION) = 6.800 PPM PER DAY
NP (MAX DO) = 12.100 PPM PER DAY
J (INITIAL DO) = 8.000 PPM PER DAY

PROBABILITY MASS DENSITIES FOR POLLUTION

| | | |
|---|---|---|
| PROBABILITY OF HAVING | 1.30 PPM | = 0.00000 |
| PROBABILITY OF HAVING | 1.40 PPM | = 0.00001 |
| PROBABILITY OF HAVING | 1.50 PPM | = 0.00002 |
| PROBABILITY OF HAVING | 1.60 PPM | = 0.00006 |
| PROBABILITY OF HAVING | 1.70 PPM | = 0.00015 |
| PROBABILITY OF HAVING | 1.80 PPM | = 0.00035 |
| PROBABILITY OF HAVING | 1.90 PPM | = 0.00076 |
| PROBABILITY OF HAVING | 2.00 PPM | = 0.00156 |
| PROBABILITY OF HAVING | 2.10 PPM | = 0.00299 |
| PROBABILITY OF HAVING | 2.20 PPM | = 0.00538 |
| PROBABILITY OF HAVING | 2.30 PPM | = 0.00910 |
| PROBABILITY OF HAVING | 2.40 PPM | = 0.01453 |
| PROBABILITY OF HAVING | 2.50 PPM | = 0.02191 |
| PROBABILITY OF HAVING | 2.60 PPM | = 0.03123 |
| PROBABILITY OF HAVING | 2.70 PPM | = 0.04215 |
| PROBABILITY OF HAVING | 2.80 PPM | = 0.05394 |
| PROBABILITY OF HAVING | 2.90 PPM | = 0.06550 |
| PROBABILITY OF HAVING | 3.00 PPM | = 0.07557 |
| PROBABILITY OF HAVING | 3.10 PPM | = 0.08288 |
| PROBABILITY OF HAVING | 3.20 PPM | = 0.08651 |
| PROBABILITY OF HAVING | 3.30 PPM | = 0.08599 |
| PROBABILITY OF HAVING | 3.40 PPM | = 0.08146 |
| PROBABILITY OF HAVING | 3.50 PPM | = 0.07359 |
| PROBABILITY OF HAVING | 3.60 PPM | = 0.06344 |
| PROBABILITY OF HAVING | 3.70 PPM | = 0.05221 |
| PROBABILITY OF HAVING | 3.80 PPM | = 0.04105 |
| PROBABILITY OF HAVING | 3.90 PPM | = 0.03084 |
| PROBABILITY OF HAVING | 4.00 PPM | = 0.02216 |
| PROBABILITY OF HAVING | 4.10 PPM | = 0.01523 |
| PROBABILITY OF HAVING | 4.20 PPM | = 0.01001 |
| PROBABILITY OF HAVING | 4.30 PPM | = 0.00630 |
| PROBABILITY OF HAVING | 4.40 PPM | = 0.00380 |
| PROBABILITY OF HAVING | 4.50 PPM | = 0.00219 |
| PROBABILITY OF HAVING | 4.60 PPM | = 0.00121 |
| PROBABILITY OF HAVING | 4.70 PPM | = 0.00064 |
| PROBABILITY OF HAVING | 4.80 PPM | = 0.00033 |
| PROBABILITY OF HAVING | 4.90 PPM | = 0.00016 |
| PROBABILITY OF HAVING | 5.00 PPM | = 0.00007 |
| PROBABILITY OF HAVING | 5.10 PPM | = 0.00003 |
| PROBABILITY OF HAVING | 5.20 PPM | = 0.00001 |
| PROBABILITY OF HAVING | 5.30 PPM | = 0.00001 |
| PROBABILITY OF HAVING | 5.40 PPM | = 0.00000 |

TOTAL PROBABILITIES = 0.98537

MEAN = 3.21 VARIANCE = 0.3539

| | | |
|---|---|---|
| PROBABILITY OF HAVING MORE THAN | 3.90 PPM | = 0.0768 |
| PROBABILITY OF HAVING MORE THAN | 3.80 PPM | = 0.1076 |

Table 13. DO Results of Segment 2 Where t = 2.50 Days

STREAM SEGMENT = 2
TIME INTERVAL = 2.500 DAYS

PROBABILITY MASS DENSITIES FOR OXYGEN

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING | 12.10 | PPM = | 0.00003 |
| PROBABILITY OF HAVING | 12.00 | PPM = | 0.00027 |
| PROBABILITY OF HAVING | 11.90 | PPM = | 0.00149 |
| PROBABILITY OF HAVING | 11.80 | PPM = | 0.00535 |
| PROBABILITY OF HAVING | 11.70 | PPM = | 0.01435 |
| PROBABILITY OF HAVING | 11.60 | PPM = | 0.03060 |
| PROBABILITY OF HAVING | 11.50 | PPM = | 0.05402 |
| PROBABILITY OF HAVING | 11.40 | PPM = | 0.08123 |
| PROBABILITY OF HAVING | 11.30 | PPM = | 0.10621 |
| PROBABILITY OF HAVING | 11.20 | PPM = | 0.12266 |
| PROBABILITY OF HAVING | 11.10 | PPM = | 0.12670 |
| PROBABILITY OF HAVING | 11.00 | PPM = | 0.11822 |
| PROBABILITY OF HAVING | 10.90 | PPM = | 0.10048 |
| PROBABILITY OF HAVING | 10.80 | PPM = | 0.07834 |
| PROBABILITY OF HAVING | 10.70 | PPM = | 0.05635 |
| PROBABILITY OF HAVING | 10.60 | PPM = | 0.03760 |
| PROBABILITY OF HAVING | 10.50 | PPM = | 0.02337 |
| PROBABILITY OF HAVING | 10.40 | PPM = | 0.01358 |
| PROBABILITY OF HAVING | 10.30 | PPM = | 0.00741 |
| PROBABILITY OF HAVING | 10.20 | PPM = | 0.00380 |
| PROBABILITY OF HAVING | 10.10 | PPM = | 0.00164 |
| PROBABILITY OF HAVING | 10.00 | PPM = | 0.00085 |
| PROBABILITY OF HAVING | 9.90 | PPM = | 0.00037 |
| PROBABILITY OF HAVING | 9.80 | PPM = | 0.00015 |
| PROBABILITY OF HAVING | 9.70 | PPM = | 0.00006 |
| PROBABILITY OF HAVING | 9.60 | PPM = | 0.00002 |
| PROBABILITY OF HAVING | 9.50 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 9.40 | PPM = | 0.00000 |

TOTAL PROBABILITIES = 0.98535

MEAN = 10.91 VARIANCE = 1.8652

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING LESS THAN | 10.80 | PPM = | 0.1454 |
| PROBABILITY OF HAVING LESS THAN | 10.70 | PPM = | 0.0891 |

Table 14. BOD Results of Segment 3 Where t = 1.0 Days

INTERMED. RESULTS (POLLUT. AND DISS.OXYGEN DENSITIES) FOR

STREAM SEGMENT = 3
TIME INTERVAL = 1.000 DAYS

ZK1 (BACTERIA COEF) = 0.120 PER DAY
ZK2 (REAERATION COEF) = 0.600 PER DAY
ZK3 (SEDIM. AND ADSORP. COEF) = 0.200 PER DAY
ZLA (LAND RUNOFF COEF) = 0.200 PPM PER DAY
ZDB (BENTHAL DEMAND) = 0.100 PPM PER DAY
ALPHA LEVEL = 0.100
I (INITIAL POLLUTION) = 10.500 PPM PER DAY
NP (MAX DO) = 12.100 PPM PER DAY
J (INITIAL DO) = 8.000 PPM PER DAY

PROBABILITY MASS DENSITIES FOR POLLUTION

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING | 2.90 | PPM = | 0.00000 |
| PROBABILITY OF HAVING | 3.00 | PPM = | 0.00000 |
| PROBABILITY OF HAVING | 3.10 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 3.20 | PPM = | 0.00002 |
| PROBABILITY OF HAVING | 3.30 | PPM = | 0.00005 |
| PROBABILITY OF HAVING | 3.40 | PPM = | 0.00010 |
| PROBABILITY OF HAVING | 3.50 | PPM = | 0.00021 |
| PROBABILITY OF HAVING | 3.60 | PPM = | 0.00042 |
| PROBABILITY OF HAVING | 3.70 | PPM = | 0.00081 |
| PROBABILITY OF HAVING | 3.80 | PPM = | 0.00147 |
| PROBABILITY OF HAVING | 3.90 | PPM = | 0.00257 |
| PROBABILITY OF HAVING | 4.00 | PPM = | 0.00428 |
| PROBABILITY OF HAVING | 4.10 | PPM = | 0.00683 |
| PROBABILITY OF HAVING | 4.20 | PPM = | 0.01045 |
| PROBABILITY OF HAVING | 4.30 | PPM = | 0.01530 |
| PROBABILITY OF HAVING | 4.40 | PPM = | 0.02150 |
| PROBABILITY OF HAVING | 4.50 | PPM = | 0.02899 |
| PROBABILITY OF HAVING | 4.60 | PPM = | 0.03752 |
| PROBABILITY OF HAVING | 4.70 | PPM = | 0.04663 |
| PROBABILITY OF HAVING | 4.80 | PPM = | 0.05569 |
| PROBABILITY OF HAVING | 4.90 | PPM = | 0.06393 |
| PROBABILITY OF HAVING | 5.00 | PPM = | 0.07057 |
| PROBABILITY OF HAVING | 5.10 | PPM = | 0.07494 |
| PROBABILITY OF HAVING | 5.20 | PPM = | 0.07659 |
| PROBABILITY OF HAVING | 5.30 | PPM = | 0.07537 |
| PROBABILITY OF HAVING | 5.40 | PPM = | 0.07142 |
| PROBABILITY OF HAVING | 5.50 | PPM = | 0.06521 |
| PROBABILITY OF HAVING | 5.60 | PPM = | 0.05738 |
| PROBABILITY OF HAVING | 5.70 | PPM = | 0.04868 |
| PROBABILITY OF HAVING | 5.80 | PPM = | 0.03982 |
| PROBABILITY OF HAVING | 5.90 | PPM = | 0.03143 |
| PROBABILITY OF HAVING | 6.00 | PPM = | 0.02394 |
| PROBABILITY OF HAVING | 6.10 | PPM = | 0.01760 |
| PROBABILITY OF HAVING | 6.20 | PPM = | 0.01249 |
| PROBABILITY OF HAVING | 6.30 | PPM = | 0.00856 |
| PROBABILITY OF HAVING | 6.40 | PPM = | 0.00567 |
| PROBABILITY OF HAVING | 6.50 | PPM = | 0.00363 |
| PROBABILITY OF HAVING | 6.60 | PPM = | 0.00224 |
| PROBABILITY OF HAVING | 6.70 | PPM = | 0.00134 |
| PROBABILITY OF HAVING | 6.80 | PPM = | 0.00078 |
| PROBABILITY OF HAVING | 6.90 | PPM = | 0.00043 |
| PROBABILITY OF HAVING | 7.00 | PPM = | 0.00023 |
| PROBABILITY OF HAVING | 7.10 | PPM = | 0.00012 |
| PROBABILITY OF HAVING | 7.20 | PPM = | 0.00006 |

Table 14. Continued

PROBABILITY OF HAVING 7.30 PPM = 0.00003
PROBABILITY OF HAVING 7.40 PPM = 0.00001
PROBABILITY OF HAVING 7.50 PPM = 0.00001
PROBABILITY OF HAVING 7.60 PPM = 0.00000
PROBABILITY OF HAVING 7.70 PPM = 0.00000

TOTAL PROBABILITIES = 0.98537

MEAN = 5.14 VARIANCE = 0.6522

PROBABILITY OF HAVING MORE THAN 5.90 PPM = 0.0918
PROBABILITY OF HAVING MORE THAN 5.80 PPM = 0.1232

Table 15. DO Results of Segment 3 Where t = 1.0 Days

STREAM SEGMENT = 3
TIME INTERVAL = 1.000 DAYS

PROBABILITY MASS DENSITIES FOR OXYGEN
PROBABILITY OF HAVING 12.10 PPM = 0.00001
PROBABILITY OF HAVING 12.00 PPM = 0.00007
PROBABILITY OF HAVING 11.90 PPM = 0.00045
PROBABILITY OF HAVING 11.80 PPM = 0.00181
PROBABILITY OF HAVING 11.70 PPM = 0.00550
PROBABILITY OF HAVING 11.60 PPM = 0.01330
PROBABILITY OF HAVING 11.50 PPM = 0.02667
PROBABILITY OF HAVING 11.40 PPM = 0.04570
PROBABILITY OF HAVING 11.30 PPM = 0.06826
PROBABILITY OF HAVING 11.20 PPM = 0.09032
PROBABILITY OF HAVING 11.10 PPM = 0.10716
PROBABILITY OF HAVING 11.00 PPM = 0.11518
PROBABILITY OF HAVING 10.90 PPM = 0.11308
PROBABILITY OF HAVING 10.80 PPM = 0.10211
PROBABILITY OF HAVING 10.70 PPM = 0.08532
PROBABILITY OF HAVING 10.60 PPM = 0.06630
PROBABILITY OF HAVING 10.50 PPM = 0.04813
PROBABILITY OF HAVING 10.40 PPM = 0.03276
PROBABILITY OF HAVING 10.30 PPM = 0.02099
PROBABILITY OF HAVING 10.20 PPM = 0.01269
PROBABILITY OF HAVING 10.10 PPM = 0.00727
PROBABILITY OF HAVING 10.00 PPM = 0.00395
PROBABILITY OF HAVING 9.90 PPM = 0.00204
PROBABILITY OF HAVING 9.80 PPM = 0.00100
PROBABILITY OF HAVING 9.70 PPM = 0.00047
PROBABILITY OF HAVING 9.60 PPM = 0.00021
PROBABILITY OF HAVING 9.50 PPM = 0.00009
PROBABILITY OF HAVING 9.40 PPM = 0.00004
PROBABILITY OF HAVING 9.30 PPM = 0.00002
PROBABILITY OF HAVING 9.20 PPM = 0.00001
PROBABILITY OF HAVING 9.10 PPM = 0.00000

TOTAL PROBABILITIES = 0.97091

MEAN = 10.61 VARIANCE = 3.4805

PROBABILITY OF HAVING LESS THAN 10.60 PPM = 0.1297
PROBABILITY OF HAVING LESS THAN 10.50 PPM = 0.0815

Table 16. BOD Results of Segment 3 Where t = 2.0 Days

INTERMED. RESULTS (POLLUT. AND DISS.OXYGEN DENSITIES) FOR

STREAM SEGMENT = 3
TIME INTERVAL = 2.000 DAYS

ZK1 (BACTERIA COEF) = 0.120 PER DAY
ZK2 (REAERATION COEF) = 0.600 PER DAY
ZK3 (SEDIM. AND ADSORP. COEF) = 0.200 PER DAY
ZLA (LAND RUNOFF COEF) = 0.200 PPM PER DAY
ZDB (BENTHAL DEMAND) = 0.100 PPM PER DAY
ALPHA LEVEL = 0.100
I (INITIAL POLLUTION) = 10.500 PPM PER DAY
NP (MAX DO) = 12.100 PPM PER DAY
J (INITIAL DO) = 8.000 PPM PER DAY

PROBABILITY MASS DENSITIES FOR POLLUTION

| | | | |
|---|---|---|---|
| PROBABILITY OF HAVING | 1.80 | PPM = | 0.00000 |
| PROBABILITY OF HAVING | 1.90 | PPM = | 0.00001 |
| PROBABILITY OF HAVING | 2.00 | PPM = | 0.00002 |
| PROBABILITY OF HAVING | 2.10 | PPM = | 0.00005 |
| PROBABILITY OF HAVING | 2.20 | PPM = | 0.00011 |
| PROBABILITY OF HAVING | 2.30 | PPM = | 0.00025 |
| PROBABILITY OF HAVING | 2.40 | PPM = | 0.00051 |
| PROBABILITY OF HAVING | 2.50 | PPM = | 0.00098 |
| PROBABILITY OF HAVING | 2.60 | PPM = | 0.00182 |
| PROBABILITY OF HAVING | 2.70 | PPM = | 0.00318 |
| PROBABILITY OF HAVING | 2.80 | PPM = | 0.00531 |
| PROBABILITY OF HAVING | 2.90 | PPM = | 0.00845 |
| PROBABILITY OF HAVING | 3.00 | PPM = | 0.01281 |
| PROBABILITY OF HAVING | 3.10 | PPM = | 0.01856 |
| PROBABILITY OF HAVING | 3.20 | PPM = | 0.02570 |
| PROBABILITY OF HAVING | 3.30 | PPM = | 0.03407 |
| PROBABILITY OF HAVING | 3.40 | PPM = | 0.04325 |
| PROBABILITY OF HAVING | 3.50 | PPM = | 0.05263 |
| PROBABILITY OF HAVING | 3.60 | PPM = | 0.06145 |
| PROBABILITY OF HAVING | 3.70 | PPM = | 0.08889 |
| PROBABILITY OF HAVING | 3.80 | PPM = | 0.07421 |
| PROBABILITY OF HAVING | 3.90 | PPM = | 0.07685 |
| PROBABILITY OF HAVING | 4.00 | PPM = | 0.07658 |
| PROBABILITY OF HAVING | 4.10 | PPM = | 0.07346 |
| PROBABILITY OF HAVING | 4.20 | PPM = | 0.06787 |
| PROBABILITY OF HAVING | 4.30 | PPM = | 0.06045 |
| PROBABILITY OF HAVING | 4.40 | PPM = | 0.08191 |
| PROBABILITY OF HAVING | 4.50 | PPM = | 0.04301 |
| PROBABILITY OF HAVING | 4.60 | PPM = | 0.03440 |
| PROBABILITY OF HAVING | 4.70 | PPM = | 0.02657 |
| PROBABILITY OF HAVING | 4.80 | PPM = | 0.01983 |
| PROBABILITY OF HAVING | 4.90 | PPM = | 0.01430 |
| PROBABILITY OF HAVING | 5.00 | PPM = | 0.00998 |
| PROBABILITY OF HAVING | 5.10 | PPM = | 0.00673 |
| PROBABILITY OF HAVING | 5.20 | PPM = | 0.00440 |
| PROBABILITY OF HAVING | 5.30 | PPM = | 0.00278 |
| PROBABILITY OF HAVING | 5.40 | PPM = | 0.00170 |
| PROBABILITY OF HAVING | 5.50 | PPM = | 0.00101 |
| PROBABILITY OF HAVING | 5.60 | PPM = | 0.00058 |
| PROBABILITY OF HAVING | 5.70 | PPM = | 0.00032 |
| PROBABILITY OF HAVING | 5.80 | PPM = | 0.00017 |
| PROBABILITY OF HAVING | 5.90 | PPM = | 0.00009 |
| PROBABILITY OF HAVING | 6.00 | PPM = | 0.00005 |
| PROBABILITY OF HAVING | 6.10 | PPM = | 0.00002 |

Table 16. Continued

PROBABILITY OF HAVING 6.20 PPM = 0.00001
PROBABILITY OF HAVING 6.30 PPM = 0.00001
PROBABILITY OF HAVING 6.40 PPM = 0.00000
PROBABILITY OF HAVING 6.50 PPM = 0.00000

TOTAL PROBABILITIES = 0.98537

MEAN = 3.90 VARIANCE = 0.4822

PROBABILITY OF HAVING MORE THAN 4.70 PPM = 0.0766
PROBABILITY OF HAVING MORE THAN 4.60 PPM = 0.1032

Table 17. DO Results of Segment 3 Where t = 2.0 Days

STREAM SEGMENT = 3
TIME INTERVAL = 2.000 DAYS

PROBABILITY MASS DENSITIES FOR OXYGEN
PROBABILITY OF HAVING 12.10 PPM = 0.00001
PROBABILITY OF HAVING 12.00 PPM = 0.00011
PROBABILITY OF HAVING 11.90 PPM = 0.00066
PROBABILITY OF HAVING 11.80 PPM = 0.00259
PROBABILITY OF HAVING 11.70 PPM = 0.00756
PROBABILITY OF HAVING 11.60 PPM = 0.01755
PROBABILITY OF HAVING 11.50 PPM = 0.03383
PROBABILITY OF HAVING 11.40 PPM = 0.05565
PROBABILITY OF HAVING 11.30 PPM = 0.07974
PROBABILITY OF HAVING 11.20 PPM = 0.10113
PROBABILITY OF HAVING 11.10 PPM = 0.11494
PROBABILITY OF HAVING 11.00 PPM = 0.11825
PROBABILITY OF HAVING 10.90 PPM = 0.11103
PROBABILITY OF HAVING 10.80 PPM = 0.09583
PROBABILITY OF HAVING 10.70 PPM = 0.07647
PROBABILITY OF HAVING 10.60 PPM = 0.05671
PROBABILITY OF HAVING 10.50 PPM = 0.03926
PROBABILITY OF HAVING 10.40 PPM = 0.02547
PROBABILITY OF HAVING 10.30 PPM = 0.01554
PROBABILITY OF HAVING 10.20 PPM = 0.00895
PROBABILITY OF HAVING 10.10 PPM = 0.00487
PROBABILITY OF HAVING 10.00 PPM = 0.00252
PROBABILITY OF HAVING 9.90 PPM = 0.00123
PROBABILITY OF HAVING 9.80 PPM = 0.00058
PROBABILITY OF HAVING 9.70 PPM = 0.00026
PROBABILITY OF HAVING 9.60 PPM = 0.00011
PROBABILITY OF HAVING 9.50 PPM = 0.00004
PROBABILITY OF HAVING 9.40 PPM = 0.00002
PROBABILITY OF HAVING 9.30 PPM = 0.00001
PROBABILITY OF HAVING 9.20 PPM = 0.00000

TOTAL PROBABILITIES = 0.97091

MEAN = 10.66 VARIANCE = 3.5038

PROBABILITY OF HAVING LESS THAN 10.70 PPM = 0.1556
PROBABILITY OF HAVING LESS THAN 10.60 PPM = 0.0989

# CONCLUSIONS

The objective of this study was to use generalized initial conditions to find the downstream probability distributions of pollution and dissolved oxygen for particular times of travel t. By use of the generalized initial conditions, the stream analysis was to have the capability of handling changes in stream parameters and the introduction of major pollution sources at several points downstream.

The results were based upon all the assumptions that the previous works of Thayer and Krutchkoff (6) and Dobbins (2) had made. In addition the assumption of independence of the two variables (BOD and DO) was necessary to obtain the DO marginal probability distribution from the appropriate joint distribution. A study of independence seemed to reveal that this assumption did not invalidate the results. A comparison with Thayer's model (no independence assumption) under the same conditions for 5 integer days illustrated that the BOD concentrations were exactly the same, and the DO concentrations were quite compatible with each other.

The final result is a computer program which can perform the arithmetic needed to find the BOD and DO probability distributions. The program handles the involved arithmetic and is very versatile in its ability to handle complex stream conditions.

The application of this study lies in the analysis of an entire stream stretch to determine if it is healthy for the users within certain preset probabilistic standards. The determination can then be made whether or not the quality of a certain polluter's effluent will be detrimental to the fish population or to the human population prior to disasterous results.

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# APPENDIX I

## COMPUTER PROGRAM WITH GENERALIZED INITIAL CONDITIONS

```
C         PROGRAM FOR FINDING PROB. DISTRIBUTIONS FOR POLLUT-
C         ION(BOD) AND DISSOLVED OXYGEN(DO) GIVEN FIXED
C         INITIAL CONDITIONS. THIS PROGRAM ALLOWS FOR CHANGES
C         IN PARAMETERS AND ADDITIONAL POLLUTION DOWN STREAM
C         BY DIVIDING THE STREAM INTO TIME INTERVAL SEGMENTS.
C
C              ( DOUBLE PRECISION )
C
      DOUBLE PRECISION  V, U, B, E, T, BA, A, Z, W, X, AI,
     1  ANP, AJ, FT, TT
      DOUBLE PRECISION UPK, WPK, F, G, Y, FE, Q, BB, BC, D,
     1  S, FQ
      DIMENSION A(500), B(500), T(500), UP(500), D(500),
     1  S(500), E(500), BA(500), FT(500), Q(500), BB(500),
     2  BC(500), FQ(500), TT(500)
      EXTERNAL NOLMUN
      READ (5, 101) KODE,TIME,ZK1,ZK2,ZK3,ZLA,ZDB,DELTA,
     1  ALPHA,I,NP,J
  101 FORMAT (I1, 8F6.0, 3I4)
      NT = 1
C        ASSIGN A PROB. TO THE INITIAL BOD AND DO
      T(I + 1) = 1.0D0
      LK = I + 1
      LKM1 = LK - 1
      EE= 10.0 ** (-15)
      DO 15 M = 1, LKM1
      B(M) = 0.0
      E(M) = 0.0
   15 T(M) = 0.0D0
      T(I + 2) = 0.0D0
C
      NJ = NP - J
      Q(NJ + 1) = 1.0D0
      LNJ = NJ + 1
      LNJM1 = LNJ - 1
      DO 139 N = 1, LNJM1
  139 Q(N) = 0.0D0
      Q(LNJ + 1) = 0.0D0
      GO TO 108
  102 IA = I
      READ (5, 101) KODE,TIME,ZK1,ZK2,ZK3,ZLA,ZDB,DELTA,
     1  ALPHA,I,NP,J
      IF (TIME.EQ.0.0) STOP
C        DETERMINE IF THERE IS ADDED POLLUTION
      IB = I
      IC = IB - IA
C        FIND AN UPPER LIMIT ON BOD PROBABILITIES
      DO 103 LKT = 1, 500
      IF (  T (LKT).GT. EE)  LK = LKT
      IF ( T(LKT).LE. EE ) T(LKT) = 0.0D0
  103 CONTINUE
      IF (IC.EQ.0) GO TO 108
C        IF BOD IS ADDED, CHANGE THE PROB ASSOC W/ STATES
      LKIC = LK + IC
      DO 104 K = 1, LK
      KK = K + IC
  104 TT(KK) = T(K)
      DO 105 KK = 1, IC
  105 TT(KK) = 0.0D0
      DO 106 KK = 1, LKIC
  106 T(KK) = TT(KK)
      LK = LKIC
```

```
C         SOLVE THE EQUATIONS
  108 AI = I
      NJ = NP - J
      ANP = NP
      AJ = J
      ZK4 = ZK2 - (ZK1 + ZK3)
      Z = (ZK1 + ZK3) * TIME
C         CHECK IF THE GIVEN PARAMETER VALUES REQUIRE THE
C         PROGRAM TO DIVIDE BY ZERO.
      IF ((Z.GT.0.0001).AND.(ZK4.GT.0.0001)) GO TO 107
      WRITE (6, 610) NT
  610 FORMAT (1H1, 5X, 20HFOR STREAM SEGMENT = I3,
     1  32H  ZK1 = ZK3 OR ZK2 = (ZK1 + ZK3) /
     2  49H REQUIRES PROGRAM TO DIVIDE BY ZERO.  SEE BOD AND
     3  9H DO EQNS. )
      GO TO 102
  107 U = DEXP(-Z)
      V = 1.0D0 - U
      W = (ZLA * V) / (DELTA * (ZK1 + ZK3))
      X = DEXP(-W)
      KA = 0
      CALL WTBIN (V, KA, U, LK + 1, B, E, T, BA)
      CALL EXPON (LK, W, LK + 1, A)
      CALL SERMU (LK, LK, BA, LK + 1, A, LK + 1, FT, LK + 1)
      IP1 = LK + 1
      AX = 0.0
      DO 114 N = 1, IP1
      BNP = N - 1
      IF (N.EQ.IP1) GO TO 115
      AX = AX +  T(N) * (BNP)
  115 UP(N) = BNP * DELTA
  114 FT(N) = X * FT(N)
      EN = 0.0
      ENN = 0.0
      DO 130 K = 1, IP1
      BNP = K - 1
      EN = EN + DELTA * BNP * FT(K)
  130 ENN = ENN + DELTA * DELTA * BNP * BNP * FT(K)
      VAR = ENN - EN * EN
      N = 1
      SUM1 = FT(1)
  111 IF (1.0 - ALPHA - SUM1) 113, 113, 112
  112 N = N + 1
      SUM1 = SUM1 + FT(N)
      GO TO 111
  113 SUM2 = SUM1 - FT(N)
      SUM3 = 1.0 - SUM1
      SUM4 = 1.0 - SUM2
      IF (KODE.EQ.0) GO TO 116
      WRITE (6, 600)
  600 FORMAT (43H1INTERMED. RESULTS (POLLUT. AND DISS.OXYGEN
     1  15H DENSITIES) FOR  )
      GO TO 117
  116 WRITE (6, 601)
  601 FORMAT (45H1RESULTS (POLLUT. AND DISS. OXYGEN DENSITIES)
     1  /, 33H AT THE END OF STREAM SEGMENT FOR  )
  117 WRITE (6, 602) NT, TIME
  602 FORMAT (1H0, 5X,  16HSTREAM SEGMENT = I3,/,6X,
     1  15HTIME INTERVAL = F7.3, 2X, 4HDAYS /)
      DI = DELTA * FLOAT(I)
      DNP = DELTA * FLOAT(NP)
      DJ = DELTA * FLOAT(J)
      WRITE (6, 603) ZK1,ZK2,ZK3,ZLA,ZDB,ALPHA,DI,DNP,DJ
```

```
  603 FORMAT (6X, 21HZK1 (BACTERIA COEF) = F7.3,2X,7HPER DAY
     1  /,6X,23HZK2 (REAERATION COEF) = F7.3,2X,7HPER DAY,/,
     2  6X, 31HZK3 (SEDIM. AND ADSORP. COEF) = F8.3, 2X,
     3  7HPER DAY/,6X,24HZLA (LAND RUNOFF COEF) = F7.3, 2X,
     4  11HPPM PER DAY,/,6X,22HZDB (BENTHAL DEMAND) = F7.3,
     5  2X, 11HPPM PER DAY,/,6X,13HALPHA LEVEL = F7.3,/,6X,
     6  23HI (INITIAL POLLUTION) = F7.3,2X,11HPPM PER DAY/6X,
     7  13HNP (MAX DO) = F7.3, 2X, 11HPPM PER DAY,/,6X,
     8  16HJ (INITIAL DO) = F7.3,2X,11HPPM PER DAY / )
       WRITE (6, 501)
  501 FORMAT (41H PROBABILITY MASS DENSITIES FOR POLLUTION)
       GT = 0.0
       DO 160 KN = 1, IP1
       IF (FT(KN) - 0.000001) 160, 160, 180
  180 WRITE (6, 502) UP(KN), FT(KN)
  502 FORMAT (23H PROBABILITY OF HAVING F7.2, 7H PPM = F7.5)
       GT = GT + FT(KN)
  160 CONTINUE
       WRITE (6, 700) GT
  700 FORMAT (1HO, 5X, 21HTOTAL PROBABILITIES = F9.5 / )
       WRITE (6, 503) EN, VAR
  503 FORMAT(8HOMEAN = F7.2, 13H  VARIANCE = F9.4/)
  185 WRITE (6, 505) UP(N), SUM3
  505 FORMAT (33H PROBABILITY OF HAVING MORE THAN F7.2,
     1    7H PPM = F7.4)
       WRITE (6, 505) UP(N-1), SUM4
C
C          CONSIDER DISSOLVED OXYGEN
C
C          FIND THE UPPER LIMIT ON DO PROBABILITIES
       DO 140 LKT = 1, IP1
       IF (Q(LKT).GT.EE) LNJ = LKT
       IF (Q(LKT).LE.EE) Q(LKT) = 0.0D0
  140 CONTINUE
C          SOLVE THE EQUATIONS
       SKT = (-ZK2) * TIME
       UPK = EXP(SKT)
       WPK = 1.0D0 - UPK
       F = (ZLA*ZK1)/(DELTA*(-ZK4))*(WPK/ZK2-V/(ZK1+ZK3))
      1 +ZDB * WPK / (DELTA * ZK2)
       G = ZK1 * (UPK - U) / (-ZK4)
       Y = 1.0D0 - G
       NP1 = NP + 1
       CALL WTBIN (WPK, KA, UPK, LNJ + 1, B, E, Q, BB)
       CALL WTBIN (Y, KA, G, LK + 1, B, E, T, BC)
       CALL EXPON (NP, F, NP1, A)
       CALL SERMU (NP, LNJ, BB, LNJ + 1, A, NP1, S, NP1)
       CALL SERMU (NP, LK, BC, LK + 1, S, NP1, FQ, NP1)
       FE = EXP(-F)
       AY = 0.0
       DO 126 N = 1, NP1
       BNP = NP1 - N
       IF (N.EQ.NP1) GO TO 125
       AY = AY +  Q(N) * (BNP)
  125 UP(N) = BNP * DELTA
  126 FQ(N) = FE * FQ(N)
       EN = 0.0
       ENN = 0.0
       DO 131 K = 1, NP1
       BNP = NP1 - K
       EN = EN + DELTA * BNP * FQ(K)
  131 ENN = ENN + DELTA * DELTA * BNP * BNP * FQ(K)
       VAR = ENN - EN * EN
```

```
      N = NP + 1
      SUM1 = FQ(N)
  118 IF(ALPHA - SUM1) 120, 120, 119
  119 N = N - 1
      SUM1 = SUM1 + FQ(N)
      GO TO 118
  120 SUM2 = SUM1 - FQ(N)
      WRITE (6, 604) NT, TIME
  604 FORMAT (1H1, 5X,  16HSTREAM SEGMENT = I3,/,6X,
     1  15HTIME INTERVAL = F7.3, 2X, 4HDAYS /)
      WRITE (6,510)
  510 FORMAT (38H PROBABILITY MASS DENSITIES FOR OXYGEN)
      GT = 0.0
      DO 170 JK = 1, NP1
      IF (FQ(JK) - 0.000001) 170, 170, 190
  190 WRITE (6, 511) UP(JK), FQ(JK)
  511 FORMAT (23H PROBABILITY OF HAVING F7.2, 7H PPM = F7.5)
      GT = GT + FQ(JK)
  170 CONTINUE
      WRITE (6, 700) GT
      WRITE (6, 512) EN, VAR
  512 FORMAT (8HOMEAN = F7.2, 13H  VARIANCE = F9.4/)
      WRITE (6, 513) UP(N-1), SUM1
  513 FORMAT (33H PROBABILITY OF HAVING LESS THAN F7.2,
     1 7H PPM = F7.4)
      WRITE (6,513) UP(N), SUM2
C
      IF (KODE.EQ.0) GO TO 195
      GO TO 102
C
C       IF THE PARAMETERS CHANGE, EQUATE THE COMPUTED PROB-
C       ABILITIES TO THE INITIAL PROBABILITIES FOR BOTH VAR-
C       IABLES AND RESOLVE.
C
  195 DO 171 KN = 1, IP1
  171 T(KN) = FT(KN)
      DO 172 N = 1, NP1
  172 Q(N) = FQ(N)
      NT = NT + 1
      GO TO 102
      END
C
C
C
C         WEIGHTED - BINOMIAL SUBROUTINE
$IBFTC WTBIN
      SUBROUTINE WTBIN (V, KK, U, M, B, C, PROB, D)
      DOUBLE PRECISION  C, B, V, AKK, AJM1, AJ, U, D, PROB,
     1  RATIO, AK
      DIMENSION B(500), C(500), D(500), PROB(500)
      IF (KK .EQ. 0) GO TO 50
C
C         CASE WHERE INITIAL POINT SOURCE IS SPECIFIED.
C
      C(1) = 1.0D0
      B(1) = V ** KK
      DO 30 J = 1, KK
      JP1 = J + 1
      AKK = KK
      AJ = J
      AJM1 = J - 1
      C(JP1) = C(J) * (AKK - AJM1) * U / AJ
      RATIO = C(JP1) / (U ** J )
```

```
   30 B(JP1) = C(JP1) * (V **(KK - J))
      RETURN
C
C        CASE WHERE INITIAL DISTRIBUTION IS SPECIFIED.
C
   50 I = 0
      D(1) = 0.0D0
      LK = M
      LKM1 = LK - 1
      DO 60 K = 1, LKM1
      J = K
   60 D(1) = (V ** K) * PROB(K + 1) + D(1)
C        THE FOLLOWING DO LOOP OF I ALSO INDEXES D STARTING
C        WITH D(I + 1)
      DO 80 I = 1, LKM1
      D(I + 1) = (U ** I) * PROB(I + 1)
      IP1 = I + 1
      LKM2 = LK - 2
      RATIO = 1.0D0
      IF (IP1.GT.LKM2) GO TO 85
      DO 80 K = IP1, LKM2
C        THE J INDEXES THE TERMS WITHIN THE K SUMMATION
C        STARTING WITH THE (0)TH TERM BEING PRESCRIBED BY
C        D(I + 1) IMMEDIATELY ABOVE.
      AK = K
      J = K - I
      AJ = J
      RATIO = (AK / AJ ) * RATIO
   80 D(I + 1) = RATIO * (V ** J) * (U ** I) * PROB(K + 1)
     I  + D(I + 1)
   85 CONTINUE
      RETURN
      END
C
C
C
C        EXPONENTIAL SUBROUTINE
$IBFTC EXPON
      SUBROUTINE EXPON (I, W, M, A)
      DOUBLE PRECISION  W, A, CK
      DIMENSION A(500)
      A(1) = 1.0D0
      DO 10 K = 1, I
      KP1 = K + 1
      CK = K
   10 A(KP1) = A(K) * W/CK
      RETURN
      END
C
C
C
C        SERIES MULTIPLICATION SUBROUTINE
$IBFTC SERMU
      SUBROUTINE SERMU (NP, I, D, MN, S, NN, T, KK)
      DOUBLE PRECISION  D, S, T
      DIMENSION D(500), S(500), T(500)
      NP1 = NP + 1
      IF (NP - I) 13, 13, 15
   13 DO 5 M = 1, NP1
      T(M) = 0.000
      DO 5 L = 1, M
      MIL = M + 1 - L
    5 T(M) = S(L) * D(MIL) + T(M)
```

```
      RETURN
   15 IP1 = I + 1
      DO 22 N = 1, IP1
      T(N) = 0.0D0
      DO 22 L = 1, N
      NIL = N + 1 - L
   22 T(N) = S(L) * D(NIL) + T(N)
      IP2 = IP1 + 1
      DO 24 K = IP2, NP1
      T(K) = 0.0D0
      I2 = K - I
      DO 24 M = I2, K
      KM1 = K - M + 1
   24 T(K) = S(M) * D(KM1) + T(K)
      RETURN
```

# APPENDIX II

## COMPUTER PROGRAM WITH FIXED POINT INITIAL CONDITIONS

```
C        THIS IS THAYERS PROGRAM (MODIFIED) FOR FINDING
C        PROBABILITY DISTRIBUTIONS FOR BOD AND DO FOR 5
C        INTEGER DAYS GIVEN AN INITIAL POINT SOURCE OF
C        POLLUTION.
C
C              ( DOUBLE PRECISION )
C
      DOUBLE PRECISION  V, U, B, E, T, BA, A, Z, W, X, AI,
     1 ANP, AJ
      DOUBLE PRECISION UPK, WPK, F, G, Y, FE, D, S
      DIMENSION A(900), B(900), T(900), UP(900), D(900),
     1 S(900), E(900), PROB(900), BA(900)
      EXTERNAL NOLMUN
      READ (5, 10) ZK1, ZK2, ZK3, ZLA, ZDB, DELTA, ALPHA, I,
     1 NP, J
   10 FORMAT (7F6.0, 3I4)
      WRITE (6, 701)ZK1,ZK2, ZK3, ZLA, ZDB, DELTA, ALPHA, I,
     1 NP, J
  701 FORMAT (43H1ZK1, ZK2, ZK3, ZLA, ZDB, DELTA, AND ALPHA
     1, 25HARE RESPECTFULLY EQUAL TO,/, 1HO, 7F6.2,/,
     2 49HOI(INITIAL POLLUT), NP(MAX DO), J(INITIAL DO) ARE,
     3 3I5)
      ITIME = 0
      AI = I
      NJ = NP - J
      ANP = NP
      AJ = J
      ZK4 = ZK2 - (ZK1 + ZK3)
   33 DO 88 NT = 1,5
      TIME = NT
      Z = (ZK1 + ZK3) * TIME
      U = DEXP(-Z)
      V = 1.0D0 - U
      W = (ZLA * V) / (DELTA * (ZK1 + ZK3))
      X = DEXP(-W)
      AA = 0D0
      CALL WTRIN (V, I, U, I+1, B, E, AA, AA)
      CALL EXPON (I, W, I+1, A)
      CALL SERMU (I, I, B, I+1, A, I+1, T, I+1)
      IP1 = I + 1
      DO 14 N = 1, IP1
      BNP = N - 1
      UP(N) = BNP * DELTA
   14 T(N) = X * T(N)
      SMEAN = (ZLA / (ZK1 + ZK3)) * V + AI * DELTA * U
      SVAR = DELTA * (ZLA / (ZK1 + ZK3) + AI * DELTA * U)*V
      N = 1
      SUM1 = T(1)
   11 IF (1.0 - ALPHA - SUM1) 13, 13, 12
   12 N = N + 1
      SUM1 = SUM1 + T(N)
      GO TO 11
   13 SUM2 = SUM1 - T(N)
      SUM3 = 1.0 - SUM1
      SUM4 = 1.0 - SUM2
      WRITE (6, 500) NT
  500 FORMAT (26H1TIME OF TRAVEL IN DAYS = I3/ )
      WRITE (6, 501)
  501 FORMAT (41H PROBABILITY MASS DENSITIES FOR POLLUTION)
      GT = 0.0
      DO 60 KN = 1, IP1
      IF (T(KN) - 0.0000001)  60,  60,  80
   80 WRITE (6, 502) UP(KN), T(KN)
```

```
502 FORMAT (23H PROBABILITY OF HAVING F7.2, 7H PPM = F7.5)
    GT = GT + T(KN)
 60 CONTINUE
    WRITE (6, 700) GT
700 FORMAT (1H0,5X,36HTOTAL PROBABILITY WITHIN THE CHART =
   1 F9.5)
    WRITE (6, 503) SMEAN, SVAR
503 FORMAT(8HOMEAN = F7.2, 13H  VARIANCE = F9.4/)
 85 WRITE (6, 505) UP(N), SUM3
505 FORMAT (33H PROBABILITY OF HAVING MORE THAN F7.2,
   1    7H PPM = F7.4)
    WRITE (6, 505) UP(N-1), SUM4
    SKT = (-ZK2) * TIME
    UPK = EXP(SKT)
    WPK = 1.000 - UPK
    F = (ZLA*ZK1)/(DELTA*(-ZK4))*(WPK/ZK2-V/(ZK1+ZK3))
   1 +ZDB * WPK / (DELTA * ZK2)
    G = ZK1 * (UPK - U) / (-ZK4)
    NP1 = NP + 1
    Y = 1.000 - G
    CALL WTBIN (WPK, NJ, UPK, NJ + 1, B, E, AA, AA)
    CALL WTBIN (Y, I, G, IP1, D, E, AA, AA)
    CALL EXPON (NP, F, NP1, A)
    CALL SERMU (NP, NJ, B, NJ + 1, A, NP1, S, NP1)
    CALL SERMU (NP, I, D, I +1, S, NP1, T, NP1)
    FF = EXP(-F)
    DO 26 N = 1, NP1
    BNP = NP1 - N
    UP(N) = BNP * DELTA
 26 T(N) = FF * T(N)
    SMEAN = DELTA * ANP - ZLA * ZK1 / ((ZK1 + ZK3) * ZK4)
   1 * V + ZLA * ZK1 / (ZK2 * ZK4) * WPK - ZDB / ZK2*WPK -
   2 DELTA * (ANP - AJ) * UPK - AI * DELTA * ZK1 / ZK4 * (
   3U - UPK)
    SVAR = DELTA * (-(ZLA * AK1)/ (ZK2 * ZK4) * WPK + ZLA
   1 * AK1 / ((ZK1 + ZK3) * ZK4) * V + ZDB / ZK2 * WPK +
   2 DELTA * (ANP - AJ) * WPK * UPK + AI * DELTA * ZK1 /
   3 ZK4 * (U - UPK) * (1.0 - ZK1 / ZK4 * (U - UPK)))
    N = NP + 1
    SUM1 = T(N)
 18 IF (ALPHA - SUM1) 20, 20, 19
 19 N = N - 1
    SUM1 = SUM1 + T(N)
    GO TO 18
 20 SUM2 = SUM1 - T(N)
    WRITE (6,500) NT
    WRITE (6,510)
510 FORMAT (38H PROBABILITY MASS DENSITIES FOR OXYGEN)
    GT = 0.0
    DO 70 JK = 1, NP1
    IF (T(JK) - 0.000001) 70, 70, 90
 90 WRITE (6,511) UP(JK), T(JK)
511 FORMAT (23H PROBABILITY OF HAVING F7.2, 7H PPM = F7.5)
    GT = GT + T(JK)
 70 CONTINUE
    WRITE (6, 700) GT
    WRITE (6,512) SMEAN, SVAR
512 FORMAT (8HOMEAN = F7.2, 13H  VARIANCE = F9.4/)
     WRITE (6,513) UP(N-1), SUM1
513 FORMAT (33H PROBABILITY OF HAVING LESS THAN F7.2,
   1 7H PPM = F7.4)
    WRITE (6,513) UP(N), SUM2
 88 CONTINUE
    STOP
    END
```

# APPENDIX III

## COMPUTER PROGRAM FOR CORRELATION COEFFICIENT STUDY

```
C
C       PROGRAM FOR DETERMINING THE DEGREE OF POLLUTION
C       (BOD) AND DISSOLVED OXYGEN (DO) IN STREAMS, AND ITS
C       DEGREE OF DEPENDENCE.
C
C       METHOD - CORRELATION COEFFICIENT
C
C       PARAMETERS    ZLA = LAND RUNOFF
C                     I = INITIAL POLLUTION (LA) / DELTA
C                     NP = CN (MAX DO) / DELTA
C                     J = CO (INITIAL DO) / DELTA
C
C       SIMPLIFIED MODEL WHERE    K3 = ZLA = DB = 0.0
C
        DIMENSION T(200), ZK1(200), ZK2(200),
       1          DELTA(200), I(200), NP(200), J(200)
        DO 30 K = 1, 200
        READ (5,20) T(K), ZK1(K), ZK2(K), DELTA(K), I(K),
       1  NP(K), J(K)
        IF (  T (K) .EQ. 0.0  ) GO TO 40
   20 FORMAT (4F5.0, 3I5)
   30 CONTINUE
C
C
   40 WRITE (6,41)
   41 FORMAT (1H1, 6X, 1HT, 7X, 3HZK1, 6X, 3HZK2, 5X,
     1 5HDELTA, 6X, 1HI, 5X, 2HNP, 6X, 1HJ, 4X, 8HBOD MEAN,
     2 5X, 7HDO MEAN, 5X, 7HBOD VAR, 6X, 6HDO VAR, 4X,
     3 9HCORR COEF, 5X, 4HCARD, / )
C
   45 DO 53 K = 1, 200
      IF (  T (K) .EQ. 0.0) STOP
      KNT = K
      A = NP(K)
      B = J(K)
      D = I(K)
      E = ZK1(K) / ( ZK2(K) - ZK1(K) )
      G = EXP( -ZK1(K) * T(K) )
      H = EXP ( -ZK2(K) * T(K) )
      XMEAN = DELTA(K) * D * G
      IF (ABS(ZK1(K) - ZK2(K) ).GT.0.0001) GO TO 46
      WRITE (6,90) T(K), ZK1(K), ZK2(K), DELTA (K), I(K),
     1 NP(K), J(K), XMEAN, KNT
   90 FORMAT (1HO, 4F9.3, 3I7, F12.5, 1X,
     1 50HUNABLE TO COMPUTE SINCE ZK1 = ZK2.  SEE VARIABLE E.
     2 2X, 14 )
      GO TO 53
   46 YMEAN = A*DELTA(K)-DELTA(K)*(A-B)*H-D*DELTA(K)*E*(G-H)
      ZMM = XMEAN * YMEAN
      CCV = D*(D-1.0)*E*G*(H-G) +A*D*G-D*(A-B)*G*H
      CCV = CCV * DELTA(K) * DELTA(K)
      XVAR = DELTA(K) * D * DELTA (K) * G * ( 1.0 - G )
      YVAR = DELTA(K) * ( DELTA(K)*(A-B)*(1.0-H)*H+D*DELTA(K)
     1        *E*(G-H)*(1.0 - E*(G-H)))
C     CC = ( CCV - ZMM ) / SQRT(XVAR * YVAR )
   50 WRITE (6,51) T(K), ZK1(K), ZK2(5), DELTA(K), I(K),
     1 NP(K), J(K), XMEAN, YMEAN, XVAR, YVAR, CC, KNT
   51 FORMAT (1HO, 4F9.3, 3I7, 5F12.5, 5X, I4)
   52  KK = KK + 1
      IF ( KK .LT. 28 ) GO TO 53
      WRITE (6,41)
      KK = 1
   53 CONTINUE
      END
```

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27 Abstract:

A stochastic model was developed and used to predict the biochemical oxygen demand (BOD) and dissolved oxygen (DO) concentrations at several points downstream of a waste discharge. The model used provided a joint density function for both pollution and dissolved oxygen with given initial conditions. The initial conditions were generalized to permit the output for one stream segment to be the initial conditions for the succeeding stream segment. Interest was focused on the measurement of the marginal distribution of BOD and DO along a stream stretch whose parameters may change at known locations. A computer program assisted in the calculations to obtain marginal distributions. The program allowed for changes in parameter values between one stream segment and the next and could predict distributions for any intermediate locations along the river. The application of this study lay in the analysis of an entire stream stretch to determine if it was healthy for the users within certain preset probabilistic standards. (Thiuri-Cornell)

Abstractor: Philip Thiuri

Institution: Cornell University

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